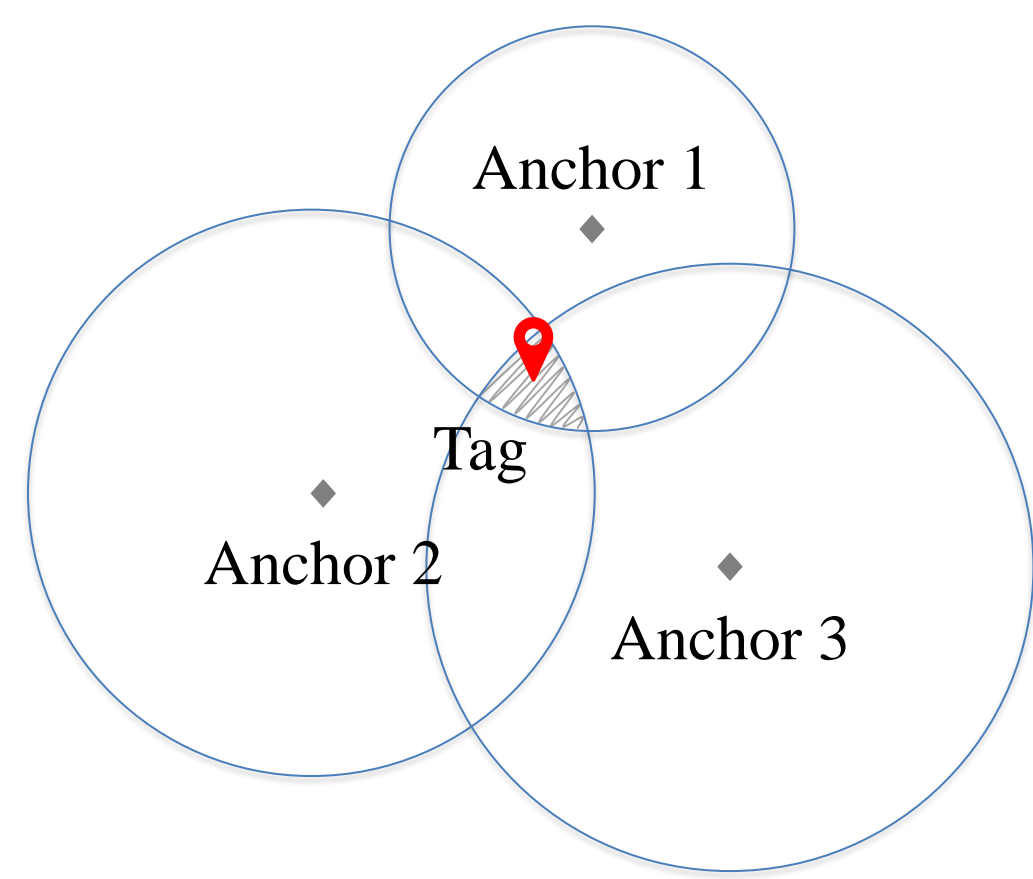


Purpose of Research

To learn the instrumental error of ultra-wideband (UWB) sensors, reduce error through data filtering, and ultimately develop an accurate positioning system.

Positioning System (PS)

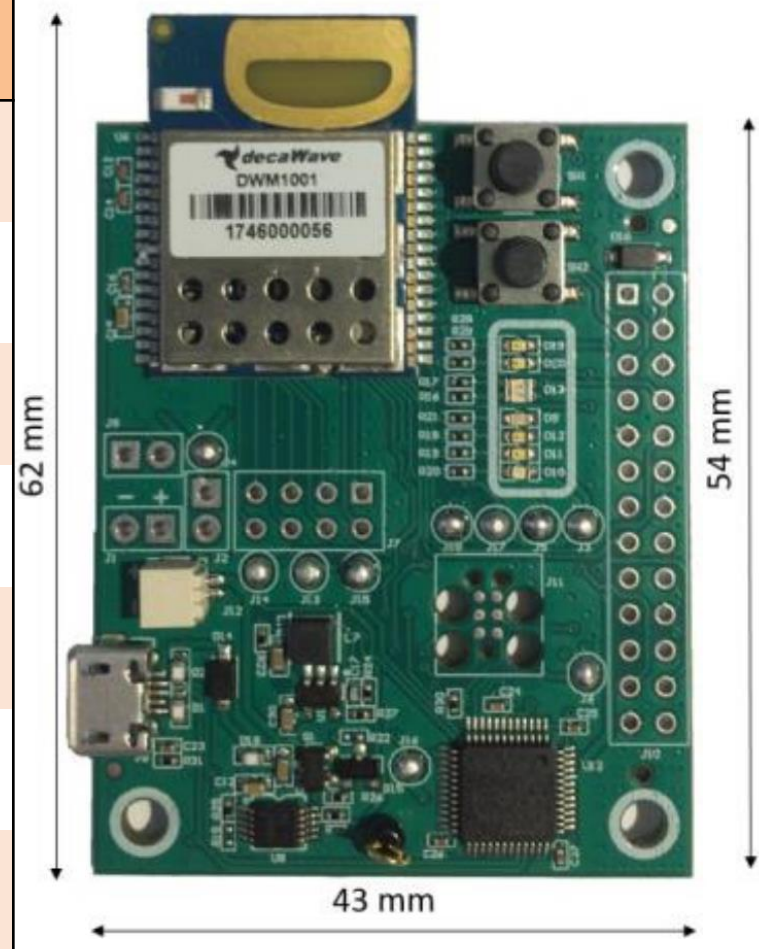


The PS discussed uses 3 UWB sensors as *anchors* to locate the 4th sensor called *tag*. Given the anchors' distance measurements from the tag and their locations, the system obtains the tag's position in x and y directions using *trilateration*.

UWB Sensor

The sensor used in this research is the Decawave DWM1001.^[1]

Specification	Value	Unit
Location Accuracy	< 10	cm
Max Range	60	m
Max Location Rate	10	Hz
Min Location Rate	0.0167	Hz
Power Supply	2.8 – 3.6	V
Data Rate	6.8	Mbps
UWB Channel 5	6.5	GHz



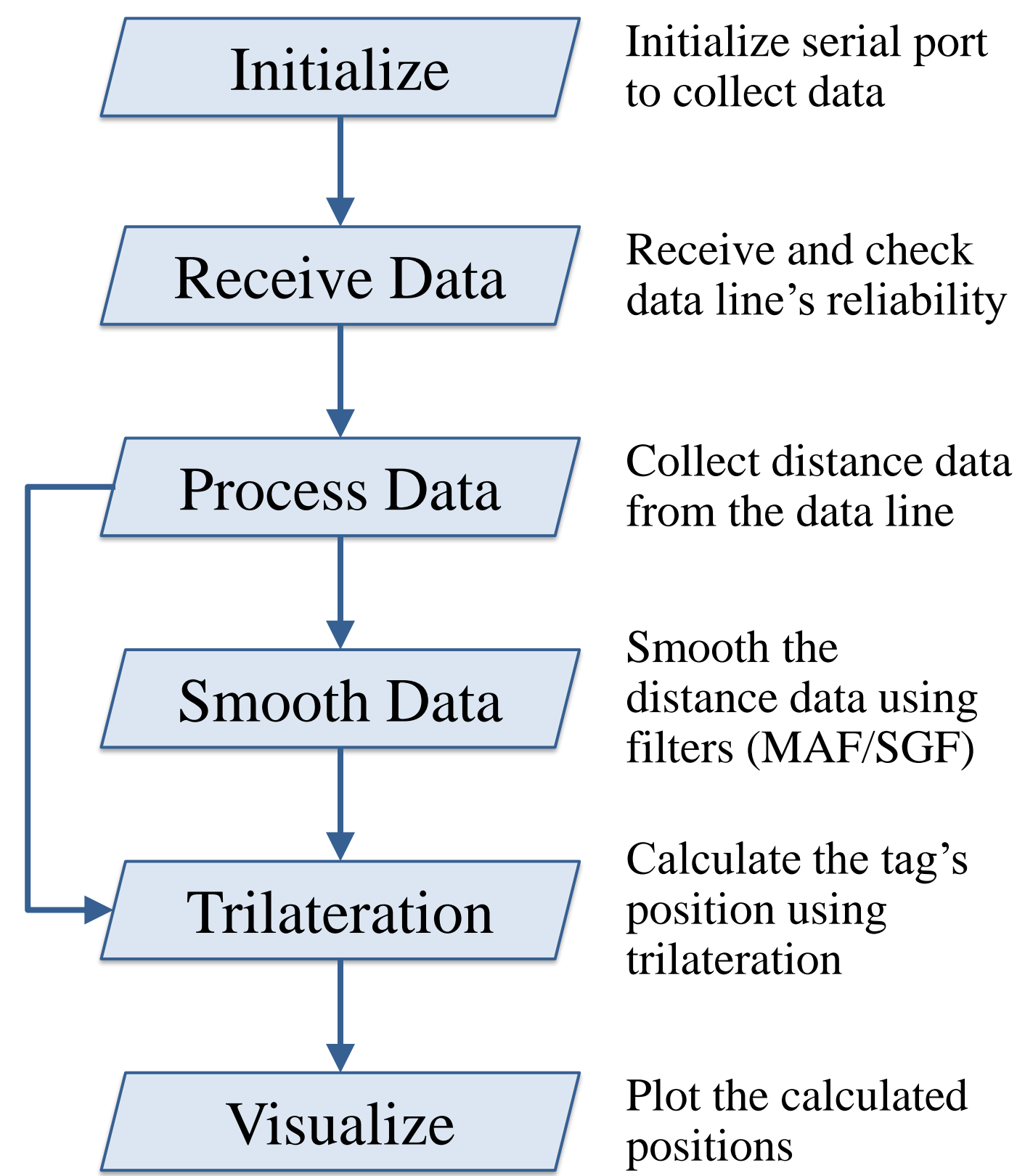
Conclusions

Both moving average and Savitzky-Golay filters show a great performance to lower the measurement error for a stationary tag localization. However, both filters are not adaptive to motions leading to time delays, which is unfavorable in real-time positioning for moving objects. A dynamic filtering approach, such as the extended Kalman filter, is planned to be considered to resolve the issues.

References

- [1] Decawave. MDEK1001 Kit User Manual Module Development & Evaluation Kit for the DWM1001, 1st ed. 2017.
- [2] D. Kim, S. Yang, and S. Lee, "Rigid Body Inertia Estimation Using Extended Kalman and Savitzky Golay Filters," *Mathematical Problems in Engineering*, Vol. 2016, pp. 1-7, Jun. 2016.
- [3] D. Kim, "Efficient Navigation for Unmanned Agents in Sparse Wireless Sensor Networks," *Transactions of Japan Society for Aeronautical and Space Sciences*, Vol. 64, No. 5, pp. 283-287, Sep. 04, 2021.

Model with Moving Average and Savitzky-Golay Filter



m	h	a_0	a_1	a_2	a_3	a_4	a_5	a_6
5	35	17	12	-3	0	0	0	0
7	21	7	6	3	-2	0	0	0
9	231	59	54	39	14	-21	0	0
11	429	89	84	69	44	9	-36	0

Moving average filter (MAF) is a method to smooth data by calculating the averages of distance measurements in successive sub-sets of adjacent data points:

$$d_t = \frac{1}{m} \left[D_t + \sum_{i=1}^{m+1} (D_{t-i} + D_{t+i}) \right]$$

where D_t is the distance measurement at time step t , and m is the number of data points in a sub-set, called window size.

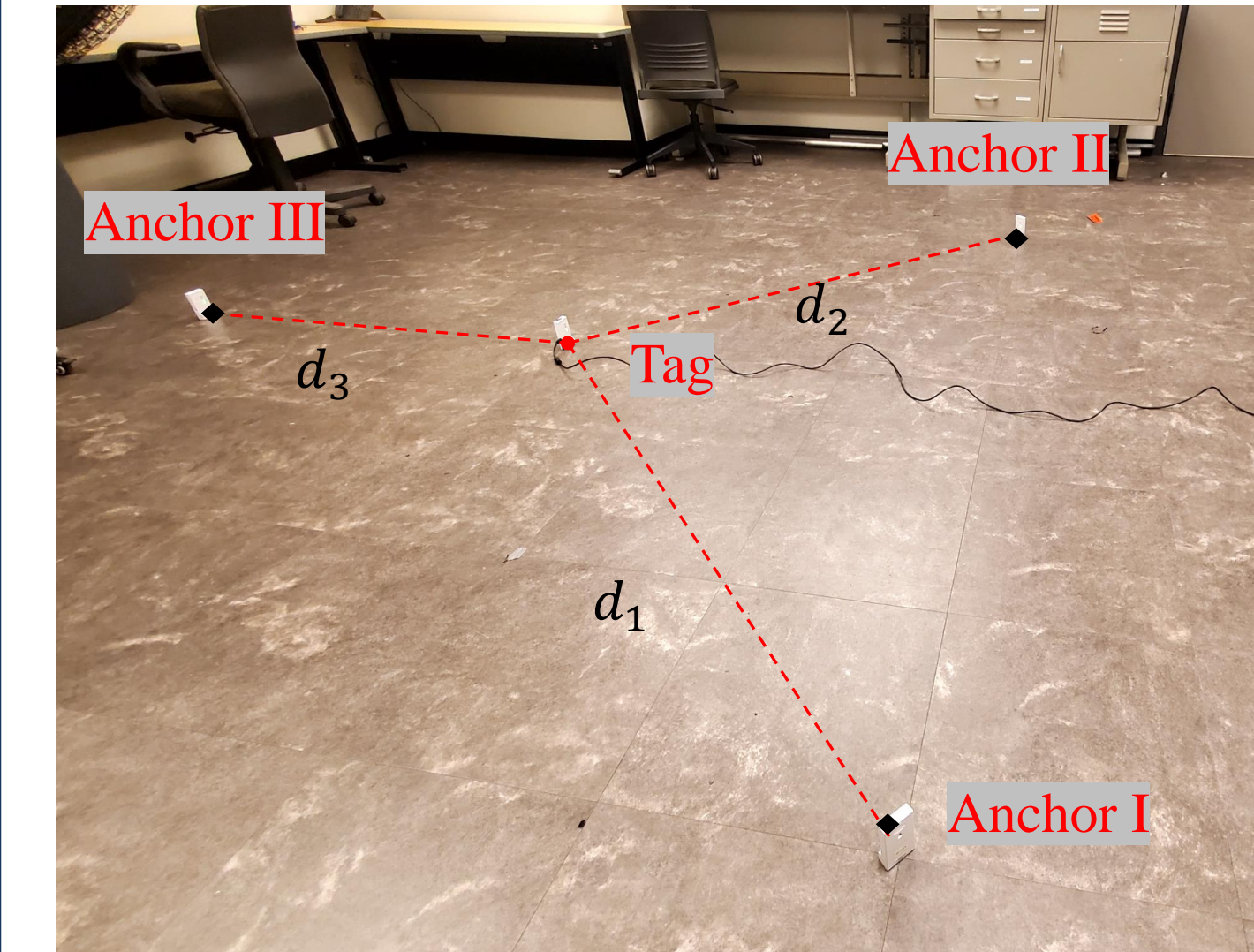
Savitzky-Golay filter (SGF) estimates the smoothed data by fitting the successive sub-sets of data with a low-degree polynomial

$$d_t = \frac{1}{h} \left[a_0 D_t + \sum_{i=1}^{m+1} a_i (D_{t-i} + D_{t+i}) \right]$$

where the normalization factor h and the "convolution coefficients" a are defined based on the window size as displayed in the table.

Experiment Settings and Trilateration

Experiment



Anchors' Position	$[x_i, y_i]$
Anchor I	[0, 0]
Anchor II	[2.72, 0]
Anchor III	[1.36, 2.72]

The distance measurements from the tag to each anchor:

$$|\mathbf{r} - \mathbf{r}_i| = d_i$$

where $\mathbf{r} = (x, y, z)$ is the tag position, $\mathbf{r}_i = (x_i, y_i, z_i)$ is the i -th anchor position, and d_i is the distance between the tag and i -th anchor.

It can be rewritten as a second order equation:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^2$$

Manipulating the equation yields the following linear equation in x and y :^[2]

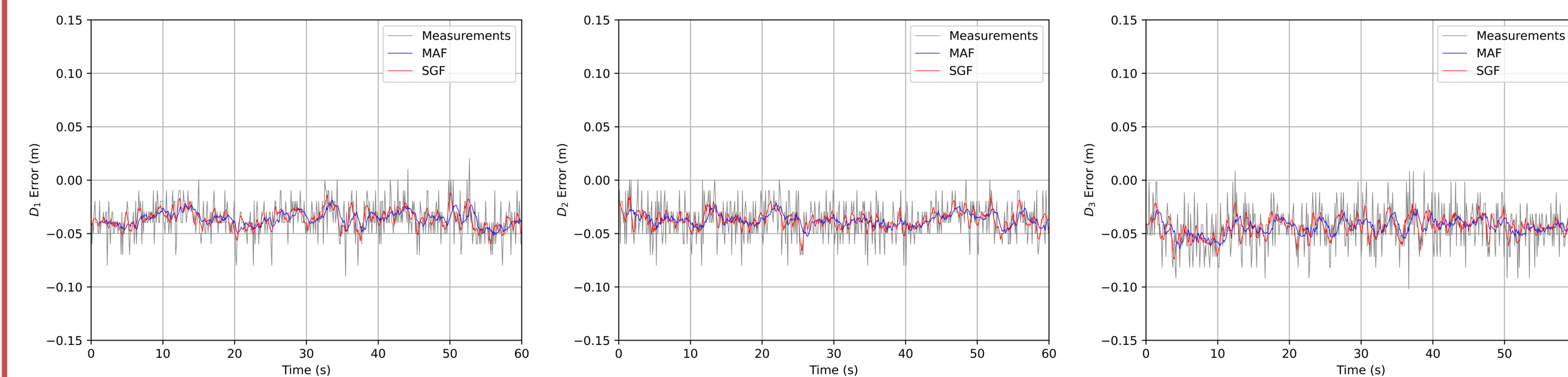
$$\begin{bmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (d_1^2 - d_0^2) - (R_1^2 - R_0^2) \\ (d_2^2 - d_1^2) - (R_2^2 - R_1^2) \\ (d_3^2 - d_2^2) - (R_3^2 - R_2^2) \end{bmatrix}$$

in which $R_i^2 = x_i^2 + y_i^2 + z_i^2$. The linear equations can be solved as

$$\mathbf{r} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$

Experimental Results of MAF and SGF Implementation

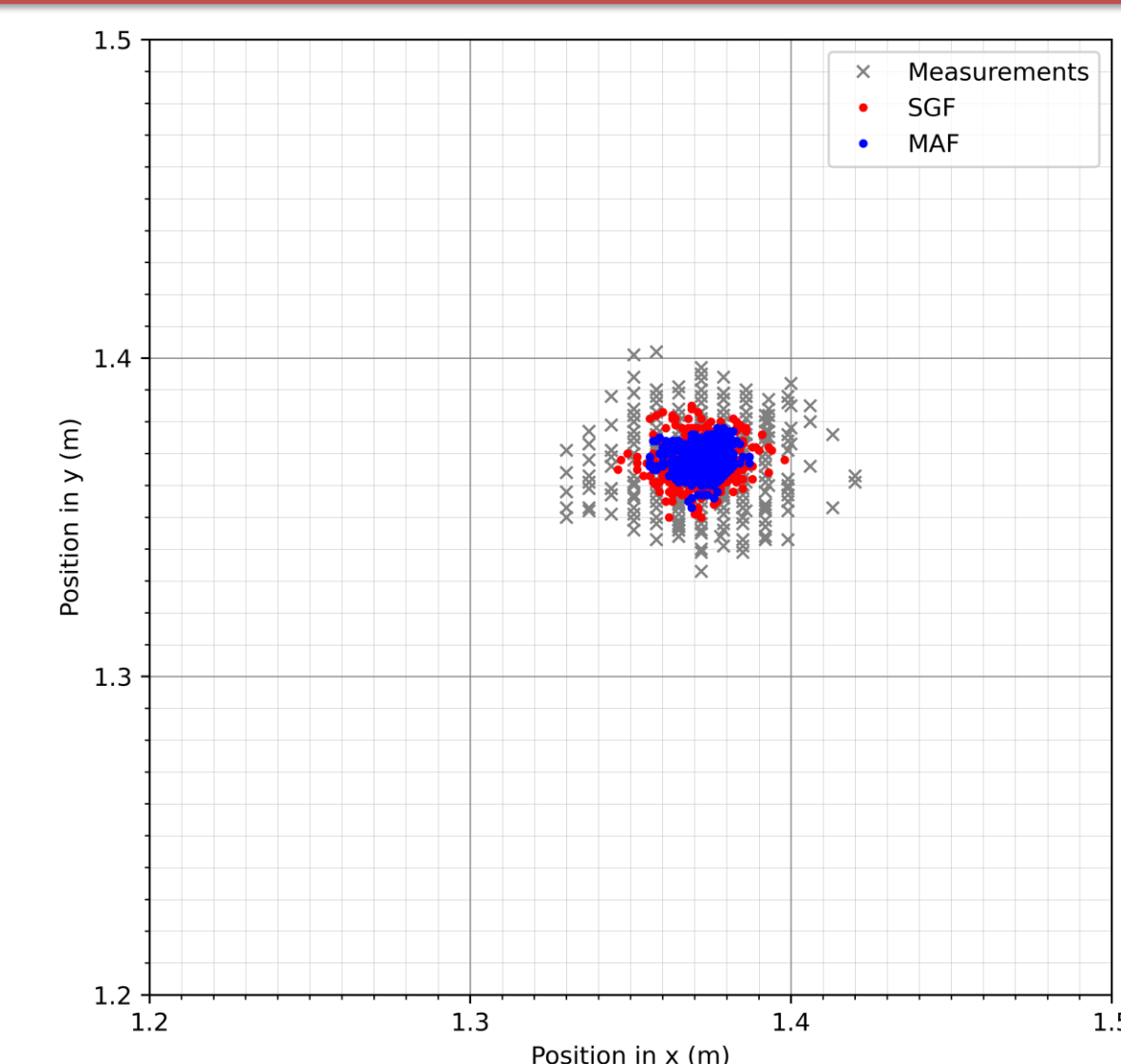
Measurements and Filtered Data



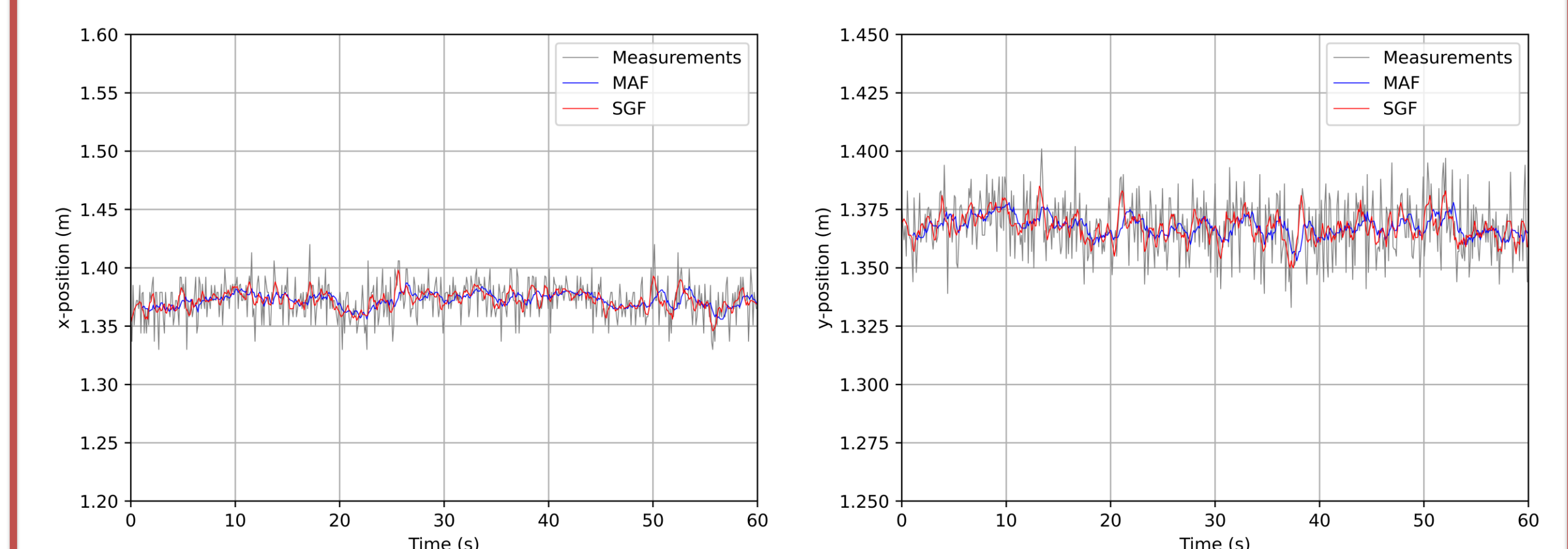
Dispersion Comparison

Standard Deviation	σ_x	σ_y
Measurements	0.01522	0.01180
MAF	0.00589	0.00421
SGF	0.00770	0.00574

- SGF increases the accuracy of the system by nearly 2 times
- MAF is around 1.5 times better than SGF, and it can increase the accuracy by nearly 3 times.
- Both MAF and SGF can cause delays in estimations.



Position of the Tag



Position Error of the Tag

