Recovering Clarity

Deblurring Images Using Linear Algebra

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Agenda

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- Causes of Blurry Images
- Theoretical Solution
- Visual Solution
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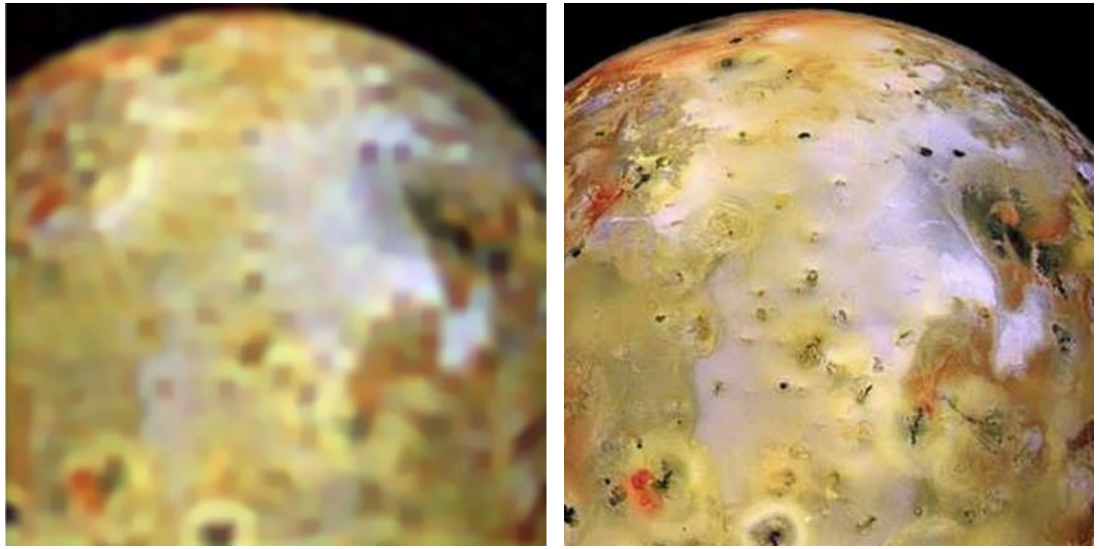
Introduction

What is Image Deblurring?

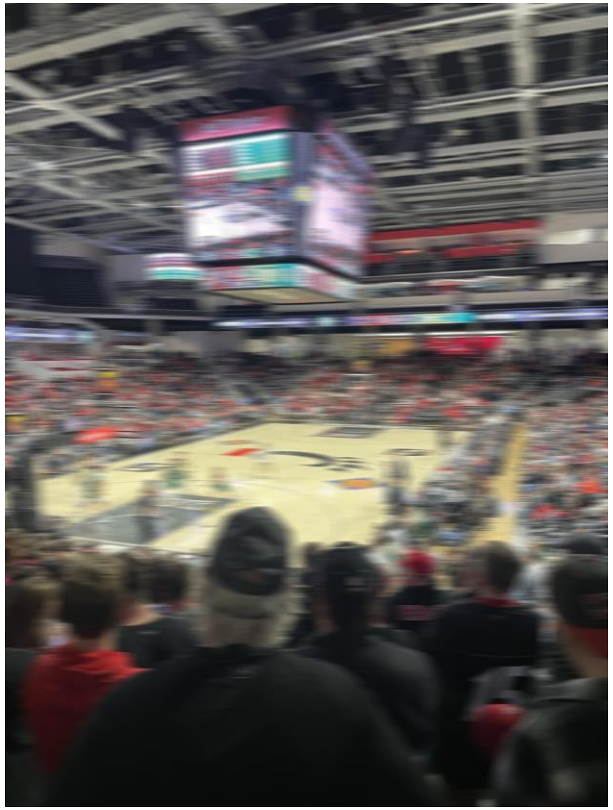
- When we capture an image, there is a chance that it can be blurry depending on the circumstances
- Image deblurring is the task of processing an image to make it a better representation of the scene- sharper and more useful.
- Deblurring is *model* based, in contrast to image enhancement techniques.

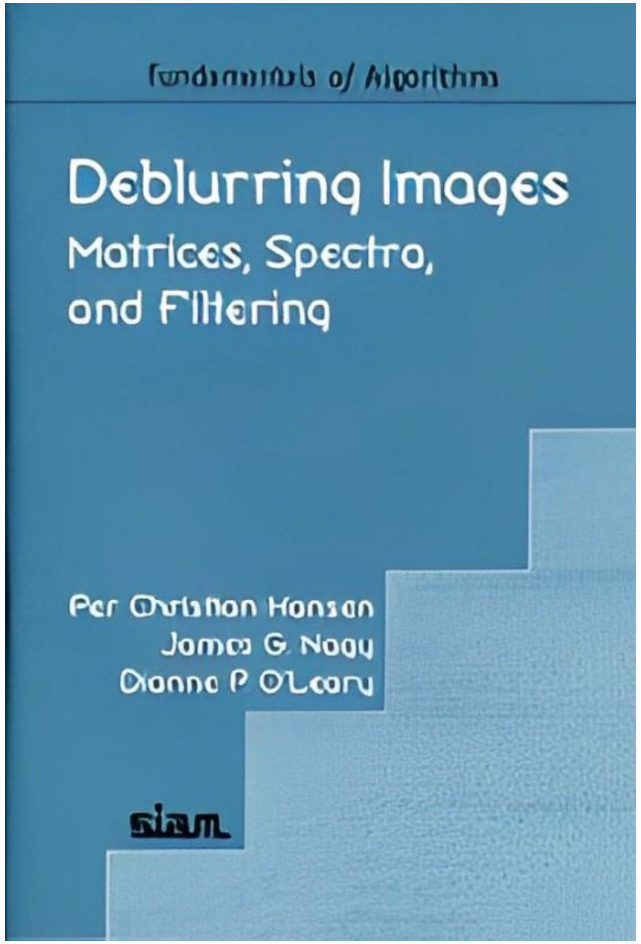


Causes of Blurry Images

- Sources of blurring:
  - Lens out of Focus
  - Camera is Shaking
  - Object is moving
  - Turbulence in Optical Path

- There are variety of blurs associated with Blurry Images such- Motion Blur, Linear Blur, Gaussian Blur





Theoretical Solution

Model/Formula Based Approach to Image Deblurring

Theoretical Solution

- For our purposes, we would be deblurring images that are **Linearly Blurred** to suit our Linear Algebra Model.
- Our Blurring Model is based on the following:

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

- Where **A** is the Blurring Matrix, **x** is the clear sharp image that we have and **b** is the blurry image obtained.

- Using this approach, in order to obtain the sharp image x again, we can do the following:

$$\mathbf{x}_{\text{naive}} = \mathbf{A}^{-1}\mathbf{b}$$

- But there are limitations to this model as it does not account for the noise **e** that is associated with linear blurring.

$$A\mathbf{x} + \mathbf{e} = \mathbf{b}$$

$$A\mathbf{x} = \mathbf{b} - \mathbf{e}$$

$$\mathbf{x} = A^{-1}\mathbf{b} - A^{-1}\mathbf{e}$$

Theoretical Solution

An Improved Method- SVD Analysis

- A method that comes to our rescue is *singular value decomposition* (SVD)
- The SVD of a matrix A is:

$$\mathbf{A} = \mathbf{U}\,\Sigma\,\mathbf{V}^T$$

- Going back to our original **Ax= b** equation, we can invert the SVD equation to obtain the following:

$$\mathbf{A}^{-1} = \mathbf{V}\,\Sigma^{-1}\,\mathbf{U}^T$$

- Matrix representation of SVD

$$\mathbf{A} = \mathbf{U}\,\Sigma\,\mathbf{V}^T = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_N \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_N \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \vdots \\ \mathbf{v}_N^T \end{bmatrix}$$

$$= \mathbf{u}_1\sigma_1\mathbf{v}_1^T + \cdots + \mathbf{u}_N\sigma_N\mathbf{v}_N^T = \sum_{i=1}^{N} \sigma_i\,\mathbf{u}_i\,\mathbf{v}_i^T .$$

- Using this the inverse of A can be:

$$\mathbf{A}^{-1} = \mathbf{V}\,\Sigma^{-1}\mathbf{U}^T = \sum_{i=1}^{N} \frac{1}{\sigma_i}\mathbf{v}_i\,\mathbf{u}_i^T .$$

Theoretical Solution

Dampening the Noise

- Now, our reconstruction solution looks like-

$$\mathbf{x} = A^{-1}\mathbf{b} - \text{ unrecoverable noise}$$

$$\text{unblurred image} = A^{-1}\mathbf{b}$$

- We can dampen the effect of noise by **truncating** the solution to only add up k matrices

- Truncation equation:

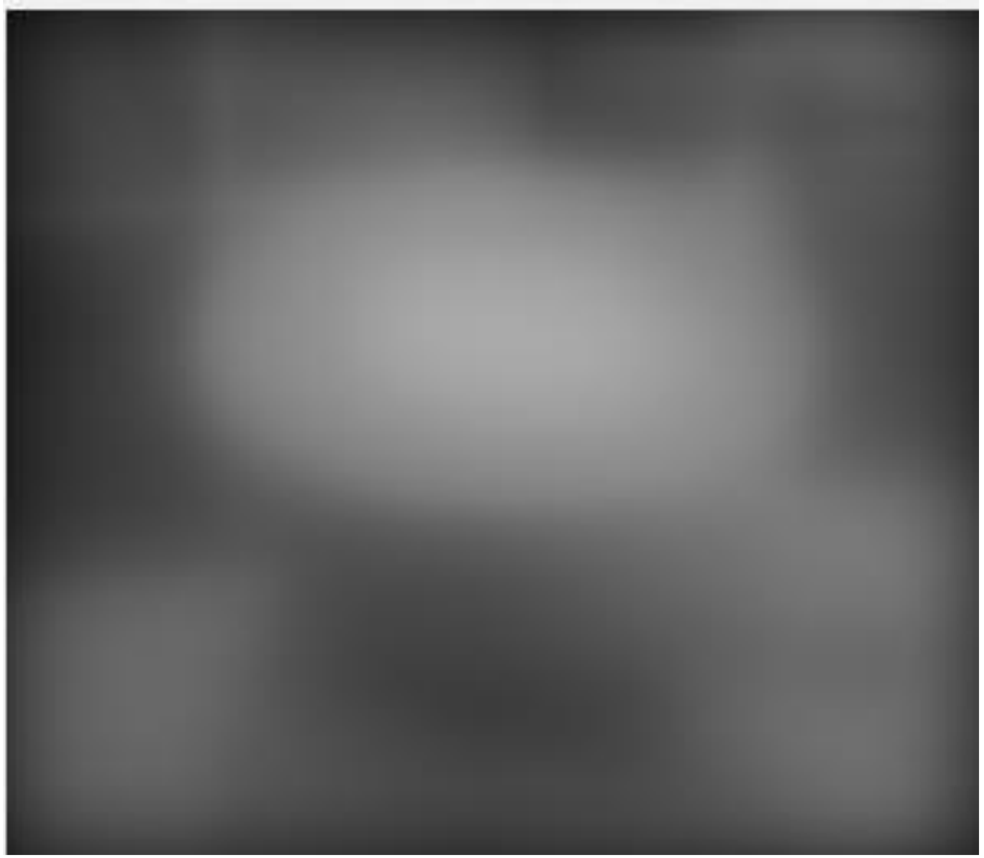
$$A^{-1}\mathbf{b} - A^{-1}\mathbf{e} = \sum_{i=1}^{N} \frac{\mathbf{u}_i^T\mathbf{b}}{\sigma_i}\mathbf{v}_i - \sum_{i=1}^{N} \frac{\mathbf{u}_i^T\mathbf{e}}{\sigma_i}\mathbf{v}_i \approx \sum_{i=1}^{K} \frac{\mathbf{u}_i^T\mathbf{b}}{\sigma_i}\mathbf{v}_i$$

Visual Solution

Using MATLAB

Any guesses on what this image might be?

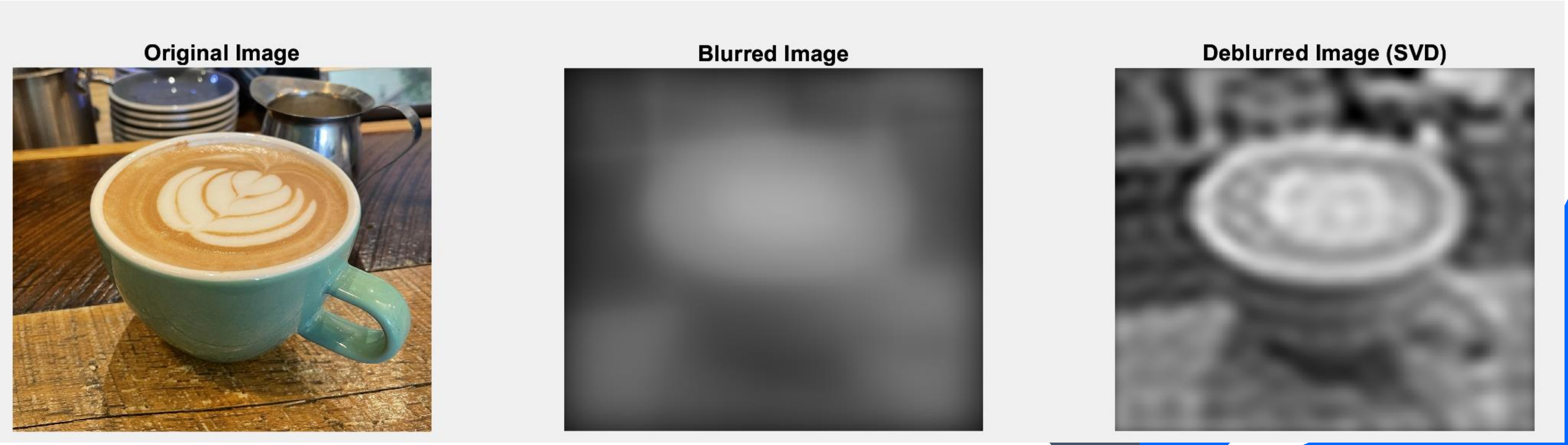
- K-values to the rescue!



Visual Solution

Using MATLAB

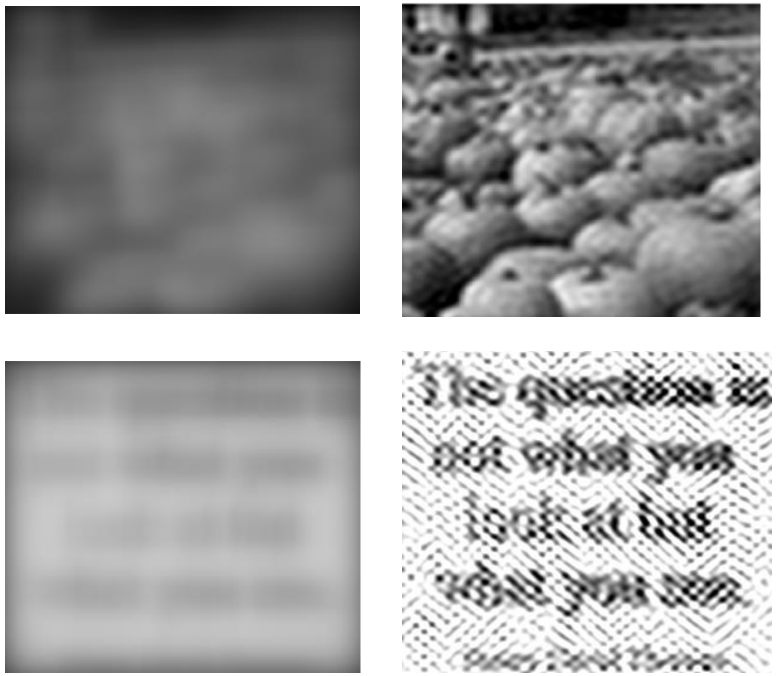
- The result you see here is the **Original Image** being **Linearly Blurred** and then **Deblurred** by implementing our **Theoretical Solution** in **MATLAB**.



Uses of Image Deblurring

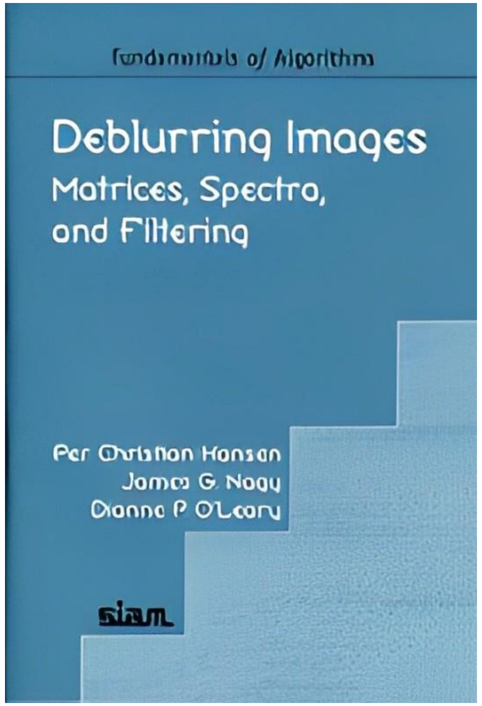
Image deblurring is useful for the following purposes:

- Deblurring our cat photos!
- Astronomical & medical images
- Bar-code readers which have to compensate for the imperfections in scanner optics



Citations

- *Deblurring Images : Matrices, Spectra, and Filtering*. (2007). Society For Industrial & Applied Mathematics, U.S.



Thank you!

Questions?