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The Fundamentals of Linear Programming: A step-by-step instructional approach

Michael Dunlop

Suffolk University

Students new to linear programming (LP) often find the problems difficult. The Linear Programming Model (LPM) is a teaching tool that is helpful in introducing students to the fundamentals of LP and how to solve basic LP problems. This tool, in the form of a blank template, condenses a basic LP problem into seven steps. The effectiveness of the LPM as a teaching tool on student learning was evaluated on an LP problem through a pre-and post-assessment. Further analysis using an independent samples *t*-test on 64 students indicates that the tool is an effective technique to improve student performance. The template is useful in organizing, analyzing, and interpreting basic LP problems and is easy to implement in the classroom.

The Linear Programming Model

George Dantzig, the father of linear programming and the inventor of the simplex method (Cottle, Johnson, & Wets, 2007), provides the foundation for optimizing business decisions. However, linear programming (LP) is often a difficult topic for instructors to teach and for students to learn. After several iterations of teaching linear programming, informal feedback from students indicated the necessity for a simple, direct, and focused approach to teaching linear programming for the students encountering it for the first time.

This study introduces the Linear Programming Model (LPM) as a useful technique for organizing, analyzing, and interpreting basic two variable LP problems, using the corner point simplex method. Similar to learning math through long-hand calculations or posting manual accounting journal entries, the LPM provides a thorough manual approach when solving LP problems. The corner point method is conceptually simpler than other LP solution techniques, such as the isoprofit line approach (Render, Stair, & Hanna Jr., 2008). Therefore, the goal of this study is to improve the student's ability to understand and apply the fundamental principles of LP.

Riddle (2010) integrated a class LP exercise that improved student performance in the three areas of decision variables, objectives, and constraint functions. Given that the application of LP is highly comprehensive, it is useful for students to be able to repeatedly practice just on the specific aspects that most interest them or where they struggle the most (Hozak, 2020). The depth to which teachers should expect students to master this particular type of modeling is often discussed from engineering to management departments (Yoder & Kurz, 2015). The LPM model offers a technique for students to practice their skills beginning with a basic corner point problem with two variables. Improving students' understanding and performance using the LPM is an effective way to introduce real world LP applications using software programs, such as MS Excel Solver. In fact, setting up a data table is the first step in the LPM, which is also similar to the first step required when using MS Excel and the Solver function. Therefore, practical teaching points are interjected throughout the completion of an LP problem.

Seven Steps of Linear Programming

The LPM includes seven useful and sequential steps. Appendix A and Appendix B provide a blank step-by-step template and a sample problem (with solution), respectively. The seven steps of teaching the fundamentals of linear programming include the following:

Step One: Construct a Data Table and Designate the Variable Expressions

The table requires students to identify the critical information necessary to organize the problem. The table in this step will lay the foundation for the table created in MS Excel.

Step Two: Identify the Objective Function and the Constraint Functions

Students are required to state problems using LP nomenclature. This step highlights the importance of staying focused on the objective of the problem, while simultaneously paying attention to the constraints. These functions are stated algebraically and serve to bridge the standard algebraic formulation for the model (Riddle, 2010).

Step Three: Find the Sets of Coordinates (X, Y) for the Constraints

Students will need to recollect basic algebraic techniques, such as the substitution method and how to summarize sets of coordinates in correct form [i.e., (X_1, Y_1) and (X_2, Y_2)].

Step Four: Plot the coordinates and draw the lines

Students become reacquainted with the Cartesian coordinate system (Quadrant I) and the concept of plotting lines and identifying corner points and lines of intersection. Furthermore, the importance of the visual LP approach is highlighted and how a well-designed graph can illustrate several important elements when determining the optimal solution.

Step Five: Corner Point Method

Therefore, when solving each of the sets of coordinates, students are encouraged to visually inspect the graphical illustration in step four with the values calculated in step five. This common-sense visual examination helps identify any obvious errors in either the sets of coordinates, the drawing of the graph, or miscalculations in the objective function. Students are redirected to the premise of the problem and what exactly is trying to be accomplished (e.g., "maximize profit"). Step five should serve as a reminder of the objective function and to organize the critical data points using the corner point method.

Step Six: Simultaneous Equations Method

Step six is only necessary if two intersecting lines meet at an undetermined coordinate. If the graph in step four is carefully constructed, then a visual estimation can be employed. However, it is critical that students use the simultaneous equations technique to determine the exact point of intersection.

Step Seven: Optimal Solution

The final step in the LPM is to summarize the optimal solution. At this point, it is recommended that students re-read the problem to ensure their understanding of the problem's objective(s) and that the correct variables are identified in form X and Y.

Post-Optimality Analysis

Students are expected to make relevant connections throughout each of the seven steps when solving the LP problem. It is critical to reinforce the fact that simply "plugging and chugging" until the optimal solution is found represents only part of the problem. The student's ability to understand the problem, explain the implications of the optimal solution, and summarize and present the results to others is of vital importance and is stressed when teaching using the LPM. Students are thus introduced to post-optimality, or sensitivity, analysis to further understand and reinforce the concepts underlying linear programming and to help in the decision-making process. For instance, students are encouraged to reflect on how changes in supply, inventory levels, and demand can all have an impact on the optimal solution. In addition, students are introduced to the concept of management's discussion and analysis (MD&A) and how to communicate the results of their analyses. For example, the Governmental Accounting Standards Board (GASB) has made the use of the MD&A a critical communication protocol. The MD&A is a narrative section that government entities present in relation to their financial statements to make it easier for a broader audience to use and interpret the financial report. The basic tenets of GASB's MD&A can be utilized by students to convey meaningful information about LP problems and the implications of post-optimality analysis. The ability to communicate with a variety of stakeholders, in both written and oral form, is a vital skill set and a valuable attribute when solving LP problems, particularly when seeking support from senior management.

Methodology and Results

To assess the effectiveness of the LPM, a two-part study was conducted. The study was conducted during the Spring 2019 academic semester. The study was conducted in two parts. Part one consisted of a small sample size used to assess the general effectiveness of the LPM as a learning tool using basic descriptive statistics. Part two consisted of a larger sample size and included a more thorough inferential statistical analysis using an independent samples *t*-test.

Part One: General Effectiveness of the LPM

Before conducting a formal statistical analysis on the effectiveness of the LPM, a small sample of twenty students was included in an in-class learning exercise. The students were enrolled in a decisions analysis course, and none of them had previous knowledge or experience working with LP. The general effectiveness of the learning exercise included the following phases of pre-assessment, intervention, and post-assessment. The phases are listed as follows:

1. Pre-Assessment: The students were given an LP problem to solve (see Appendix B) without introducing the LPM "Step-by-Step Template" (see Appendix A). Students were given 15 minutes to solve the problem.
2. Intervention: The students were tasked with a different LP problem, but similar in scope. For this problem, students were introduced to the LPM "Step-by-Step Template" and how this tool can be used to organize and solve basic LP problems. This phase is the learning intervention with students learning how to use the template.
3. Post-Assessment: Students were once again asked to revisit and solve the initial problem from Appendix B. This time they were allowed to use the LPM "Step-by-Step Template." Students were given 15 minutes to solve the problem. The solution to the problem in Appendix B using the LPM "Step-by-Step Template" is illustrated in Appendix C.

Results from the learning exercise indicate that the LPM is an effective tool for students as they learn how to solve LP problems. Table 1 summarizes the results of the learning exercise. The first column of the table indicates the three phases of the learning exercise conducted in class. The next three columns summarize the number of students who answered the problem incorrectly, partially correctly, or correctly. To accurately assess student performance, the instructor in the class visually inspected each student's work to determine whether the problems were incorrect, partly correct, or correct. Incorrect, partly correct, and correct were qualified as follows:

Incorrect: The student did not get past step one of the LPM.
Partly Correct: The student completed the table in step one and defined the objective and constraint functions in step two. However, the student progressed no further or answered all the other steps incorrectly.
Correct: The student successfully completed all six steps, with the correct answer summarized in step 7.

Table 1

Summary of Student Performance on LPM Exercises

Phase	Incorrect	Partially Correct	Correct	Total (n=20)	% Correct
I. Pre-Assessment: Appendix B Problem without Appendix A Template	15	5	0	20	0%
II. Intervention: Different Problem with Appendix A Template	2	9	9	20	45%
III. Post-Assessment: Appendix B Problem with Appendix A Template	0	3	17	20	85%

The last column of the table indicates the percentage of students able to solve the problem correctly. Analysis of the results suggest that the LPM tool is an effective technique that improves student learning. This part of the study was simply an analysis of the change in percentages between the pre-assessment and post-assessment, respectively. No further statistical analysis was performed. An informal poll conducted at the end of the class indicated that an overwhelming majority of students (over 96%) found the LPM template a helpful learning tool. In fact, several students inquired if the LPM template would be allowed during the next in-class examination.

Part Two: Statistical Test of the LPM

Based on the assumption that the LPM was an effective learning technique, a larger sample size (n = 64) of new students participated in the second part of the study. The students were unfamiliar with LP and were enrolled in decisions analysis and operations management courses. Students were first introduced to the fundamental theoretical underpinnings of linear programming and were given a problem to solve with the instructor's assistance. Initially the problem was solved without the aid of the LPM template. Afterwards, the same problem was solved using the LPM template. The next step was to segment the students into two smaller groups (n = 32) and to administer a different, yet similar two variable LP problem. To assess the students' performance, a 10-question quiz was administered. The quiz was graded on a 100-point scale (10 points per question). Students were informed that the quiz would only count as a participation grade (regardless of how many questions they answered correctly). Students were reminded that this was simply a teaching exercise to prepare them for the final exam.
Group one (μ_1) was not provided with the LPM template, while group two (μ_2) was provided with the LPM template.

An alpha level of 0.05 was selected. An analysis of the results indicated that the students who were in group two, using the LPM template, performed at a higher level than group one, which did not use the LPM template. The difference between the two groups was statistically significant as indicated by the small *p*-values shown in Table 2.

Table 2
Independent Samples t-Test of Student Quiz Performance

	Group One	*Group Two*
Mean	65.3125	83.125
Variance	586.9959677	338.3064516
Observations	32	32
Pooled Variance	462.6512097	
Hypothesized Mean Difference	0	
df	62	
t Stat	-3.312516252	
P(T<=t) one-tail	0.000772996	
t Critical one-tail	1.669804163	
P(T<=t) two-tail	0.001545993	
t Critical two-tail	1.998971517	

Conclusion

Incorporating teaching techniques and aids, such as the Linear Programming Model (LPM) template, are valuable in the student learning process. Helping students make logical connections between theory and practical applications is beneficial to the learning environment. In this study, students who solved LP problems using the LPM template strongly outperformed those who did not use the template. Results from this study support the effectiveness of the LPM template as an effective and useful tool for students learning how to solve basic LP problems and by preparing them for more complex LP problems, such as sensitivity analysis and multivariable LP.

A limitation in this study is the small sample size. The LPM template could be further tested to determine its effectiveness in online teaching environments as well as using software, such as the MS Excel Solver function. Given that LP is often intimidating to students learning the topic for the first time, the LPM framework offers a systematic and logical approach. This approach appears to be successful based on the results of the study and the statistically significant differences between the independent samples. Since this introductory exercise in LP can be completed in approximately 30-45 minutes, it can be a valuable tool for other instructors to incorporate in their classrooms.

References

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APPENDIX A: Step-by-Step Template

Step by Step - LP Solution Outline

Step 1: Construct a Data Table & Designate the Variable Expressions

Step 2: Identify the Objective Function & the Constraint Functions

Objective Function

Maximize Profit: ____________________________

Minimize Cost: ____________________________

Subject to the following constraints (name each):
1) ______________ Constraint: ____________________________

2) ______________ Constraint: ____________________________

3) ______________ Constraint: ____________________________

4) Non-Negativity Constraint: ____________________________

Step 3: Find the Sets of Coordinates (X, Y) for the Constraints

First Constraint:	Second Constraint:	Third Constraint:
When X=0, Y=	When X=0, Y=	When X=0, Y=
When X= , Y=0	When X= , Y=0	When X= , Y=0
Coordinates:	**Coordinates:**	**Coordinates:**

Step 4: Plot the coordinates and draw the line

Step 5: Corner Point Method

Identify the extreme points from Step 4. Keep in mind the non-negativity constraint.

Point	**Simultaneous Equation**	**Coordinates**	**Profit/Cost Function** *Formula =*
	(See Step 6(if necessary)		
Point ___		X = , Y =	
Point ___		X = , Y =	
Point ___		X = , Y =	
Point ___		X = , Y =	

Step 6: Simultaneous Equations Method

Point ___

Point ___

Point ___

Step 7: Optimal Solution

X =	**Y =**	**Profit/Cost = $**

APPENDIX B: Sample Problem

Penelope's Bakery makes scones and coffee cakes in large baking pans every day. The main ingredients are sugar and flour. There are 50 pounds of sugar and 70 pounds of flour available. Four pounds of sugar and five pounds of flour are required to make a pan of scones. A pan of coffee cakes requires three pounds of sugar and five pounds of flour. A pan of scones generates a profit of $9.00 and a pan of coffee cakes generates a profit of $8.00. Based on the above information, determine the optimal number of pans of scones and pans of coffee cakes to produce each day so that profit will be maximized.

APPENDIX C: Sample Problem (with Solution)

Step by Step - LP Solution Outline

Step 1: Construct a Data Table & Designate the Variable Expressions

_	Scones	Coffee Cakes	Amount Available
Sugar	4	3	50
Flour	5	5	70
Profit	$9.00	$8.00	_

Step 2: Identify the Objective Function & the Constraint Functions

Objective Function

Maximize Profit: $9.00X + $8.00Y

Minimize Cost: Not Applicable (N/A)

Subject to the following constraints (name each):

2) Sugar Constraint: $4X + 3Y \leq 50$

2) Flour Constraint: $5X + 5Y \leq 70$

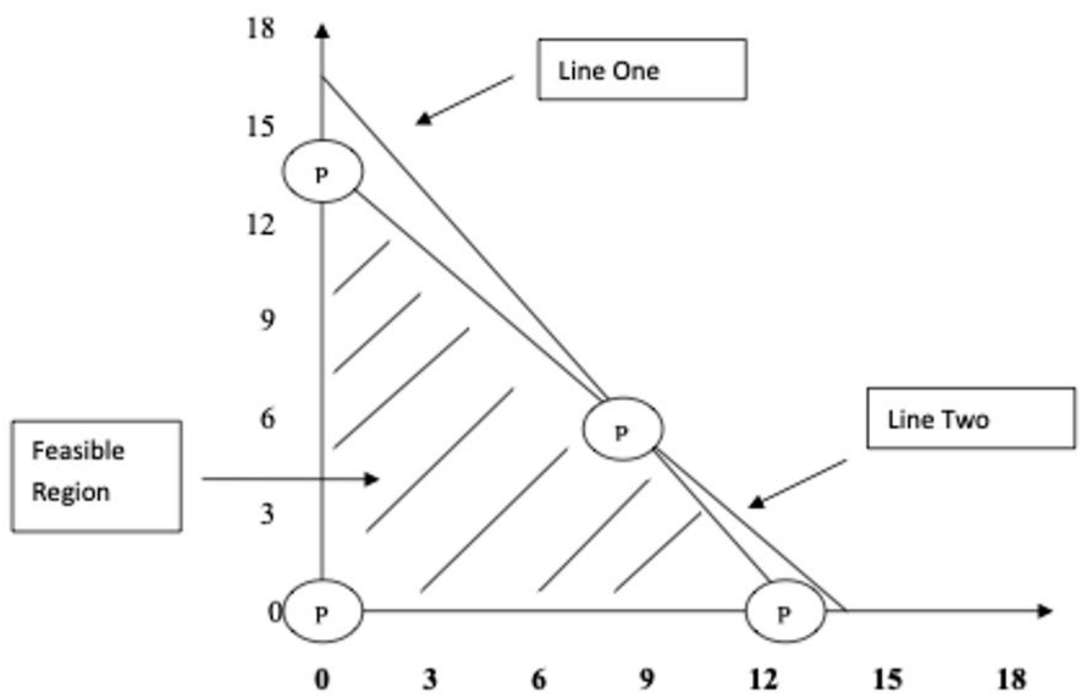
3) N/A Constraint:

4) Non-Negativity Constraint: $X, Y \geq 0$

Step 3: Find the Sets of Coordinates (X, Y) for the Constraints

First Constraint: 4X + 3Y = 50	Second Constraint: 5X + 5Y = 70	Third Constraint: N/A
When X=0, Y= 16.67	When X=0, Y= 14	When X=0, Y=
When X= 12.5 , Y=0	When X= 14, Y=0	When X= , Y=0
Coordinates: **(0, 16.67) (12.5, 0)**	**Coordinates:** **(0, 14) (14, 0)**	**Coordinates:**

Step 4: Plot the coordinates and draw the lines



Step 5: Corner Point Method

Identify the extreme points from Step 4. Keep in mind the non-negativity constraint.

Point	Simultaneous Equation	Coordinates	Profit/Cost Function Formula =
	See Step 6(if necessary)		
Point __1_		X = 0 Y = 0	\$0.00
Point __2_		X = 0 Y =14	\$112.00
Point __3_	step 6	X = 8 Y = 6	\$120.00
Point __4_		X = 12.5 Y = 0	\$112.50

Step 6: Simultaneous Equations Method

Point _3__ L_1: 5 (4X + 3Y = 50)

L_2: -3 (5X + 5Y = 70)

Rewritten:

L_1: 20X + 15Y = 250

L_2: -15X - 15Y = -210

Therefore: X = 8 and Y = 6

Point _N/A__

Point _N/A__

Step 7: Optimal Solution

X = 8 Scones	Y = 6 Coffee Cakes	Profit/Cost = \$ 120.00