Geometries Past, Modern, and Future:
Developing a rigorous approach to complex form through historical techniques of geometric description

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This paper presents the work of Arch201, a second semester undergraduate design studio at the University of Pennsylvania. But before delving into the work produced by the studio, it’s important to say a bit more about the program itself, as it’s somewhat unusual. At Penn, the undergraduate architecture program is not situated at the School of Design with our graduate program, but rather within the School of Arts and Sciences. This has impacted the curriculum in a variety of ways, which will be explained in a moment. The undergraduate program offers both a Major and Minor in architecture. The degree earned via the major is a Bachelor of Arts (B.A), but there are also several subdivisions within the major: a concentration in History, Theory, and Criticism; a concentration in Design; and what we call an Intensive Major with concentration in Design (figure 1).

The curriculum for the concentration in History, Theory, and Criticism requires four studios, two architectural history and theory courses, and six courses in the History of Art. The concentration in Design requires six studios, two architectural history and theory courses, and four courses in the History of Art. And the Intensive Major with concentration in design includes the first-year technology courses from PennDesign’s MArch program. What’s notable, relative to the typical undergraduate curriculum, is the lack, or reduction, of courses in subjects like representation, structures, environmental systems, construction, and so on. This is where the program’s situation within the within the School of Arts and Sciences becomes important.

The stated mission of the undergraduate program in Architecture is to develop basic skills, knowledge, and methods of inquiry in the discipline of architecture within the context of a liberal education in the arts and sciences. To accomplish this, efforts are made to allow students to broaden the scope of their studies by taking courses in other subjects and departments. In fact, the majority of courses students take are left to their own selection rather than being dedicated to core subjects in architecture, as is typical. Related to this, is that a large portion of our students, typically between 25-40%, double major, pairing their Bachelor of Arts in architecture with degrees in fields like Urban Studies, Engineering, Biology, Physics, Philosophy, Visual Studies, Fine Arts, and Music, to name just a few. Nearly all of our students declare one or more minors. The degrees we offer in architecture are not professional degrees, but our graduates are equipped with a foundation of design skills and a broad and rigorous intellectual training in liberal arts. Most our graduates do eventually pursue an MArch.

Curricular Orientation

Over the last five years, the undergraduate program’s chairman, Richard Wesley, has increasingly focused the curriculum on questions of geometry. This has been based on what he calls “a critical appreciation of contemporary digital architecture—neither an uncritical acceptance nor an unappreciative critique.” Over the past few decades, tools for flexible surface modeling (such as Rhinoceros 3D) and visual scripting (such as Grasshopper) have offered designers the opportunity produce and manipulate complex forms and double curved surfaces with ease. These tools operate by transforming static geometry into parametric models, sets of geometric relationships. However, as Mark
Gage has written, in providing designers with “easy access to the difficult processes of computation” these tools suffer from their own inherent biases and obfuscate the range of geometric procedures that underlie the production of such complex forms. The software acts as a black box. To counter this, Penn’s undergraduate program aims to equip students with techniques and ways of understanding geometry (in both two and three dimensions) which allow them to better control these digital tools and approach the production of complex form critically, with precision and control. In our program, “freeform” manipulation of geometry is strictly forbidden.

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*Figure 1. Curricular structure for the three concentrations.*

Arch 102, the first course in our studio sequence, begins in the second semester of students’ freshman year. This course is an architectural skills primer, meant to provide students with a range of experiences in design and help them decide if a degree in architecture is right for them. The course meets for just two hours, two days a week. Students visit sites in the interior of buildings on Penn’s campus, which they measure and draw (figure 2). They then intervene within the space to design a reading carrel. Projects are generally orthogonal, and are developed through sketch, drafting in AutoCAD, and sketch model. Final models are constructed with laser-cut MDF.
The second course in design, Arch 201, focuses on the analysis and geometric description of more complex forms, many of them curved. We analyze the geometries of biological organisms (figure 3), and through them develop methods of describing their formal properties, qualities, and characteristics through drawing (figure 4). We next use these same methods to generate architectural prototypes (figure 5-6), which are then adapted and reinterpreted within a specific site. This work will be presented in depth below. We also run studio workshops to help students learn how to use Rhino, Illustrator, and Photoshop.
Figure 4. Geometric analysis of Pink Waxcap fungus by Xinping Yang.

Figure 5. Prototype by Xinping Yang, as based on Pink Waxcap fungus.
Figure 6. Geometric process for generating prototype by Xinping Yang, as based on Pink Waxcap fungus.
Arch 202, in the second semester of the sophomore year, involves the geometric analysis of a bodily movement or activity. This is then used to inform the program and conceptual strategy for intervening within a strictly bound condition—a gutted Philadelphia rowhouse. This studio is currently under revision.

Arch 301, in the first semester of the junior year, introduces students to the use of Grasshopper which they first use to design a series of geometric patterns. Student’s pattern studies are then folded into the manipulation of geometric primitives, which in turn inform the design of an artists gallery and studio—a multi story building at the end of a row, with the project including the design of the building’s envelope (figure 7).

The final two studios of the sequence are often in flux but have focused on varied sorts of projects including a boathouse on a river in Philadelphia, a monastery in rural Pennsylvania, a museum on campus, and a school in Ghana. Issues dealt with in these studios have ranged from the development of triangulated structures and the design of complex space trusses, to spatial narrative and a building’s situation within landscape.

A Pedagogical Bridge

Within the scope of the course sequence I’ve described, it’s the charge of the second year in the curriculum to bridge between the orthogonal spatial compositions of the Arch 102 reading carrels and the computational focus of the Arch 301 pattern studies with Grasshopper. As I’ve stated, we begin the semester with the geometric analysis of biological forms. We first ask the students to choose an organism to study and some specific aspect of its form on which to focus—like particular anatomical features, its structural or organizational principles, or even patterns of growth. Examples of the sorts of things students have chosen are: the structure of a Cactus skeleton, the carapace of a snow crab, the curvature and proportioning of a praying mantis’ body segments, the gill structure of the pink waxcap fungus, the curvature of manta ray fins, or the rhythm and structure of a deer fern’s leaves (figure 8).

We begin by asking students to take an image of the organism which displays the feature to be analyzed, taking care to avoid perspectival distortions and approximate an elevational view. Students then, in the spirit of drawings by d’Arcy Wentworth Thompson (figure 9), overlay the image with an orthogonal gird, plotting points on the grid in attempt to describe the form. This method, by design, proves insufficient, or at least sub-optimal. Just as Thompson deformed his own grids to describe the geometries of biological growth and change, we then ask students to deform their overlaid grids by
changing sets of variables in order to better reveal the underlying geometries of the form studied, and once again plot points within the network of lines to describe the form (figure 10). From this sequence of drawings, we then ask each student to develop their own methods for the geometric analysis of their organism. As a springboard for this next phase, we present to the students and discuss with them a range of historical drawings regarding the classical orders, stone cutting, and descriptive geometry.

Figure 8. Examples of organisms studied.

Figure 9. Images from Thompson’s On Growth and Form.
Figure 10. Jellyfish analysis, grid overlay and series of adjustments by Galena Sardemova.

Figure 11. Drawings from Francois Blondel’s Cours d’architecture (1675) illustrating the geometric and proportional composition of the Corinthian order.

Figure 12. Device for drawing a hyperbolic line (right) and geometric principles of its operation (left) from Francois Blondel’s Cours d’architecture (1675).
Figure 13. Geometries for stereotomy, as published in A. Frezier’s *La théorie et la pratique de la coupe des pierres* (1737).

Figure 14. Geometric drawings from Gaspard Monge’s *Géométrie descriptive* (1799).
We began with the 17th century drawings of Francois Blondel—highlighting their composition of multiple views, analytic dissection of elements, revelation of underlying geometries, use of varied line types and weights, and nesting of notation and proportional information within the drawing (figure 11). We also discussed Blondel’s drawings of drafting instruments, which juxtaposed representations of the objects themselves with geometric descriptions of their use and performance (figure 12). Next were drawings by Amadee-Fancois Frezier, from his 1737 treatise on the theory and practice of cutting stone and wood. Frezier’s drawings offered us again examples of notating drawings and the use of varied sorts of lines, but more importantly they illustrated increasingly complex modes of geometric description (figure 13). And of these methods, ways of drawing and analyzing curvature were of particular interest. (Pause to move through images) From Frezier we moved on to the late 18th and early 19th century publications of Gaspard Monge, the famed French educator of architects and engineers (figure 14). From his drawings we adopted a variety of techniques for analyzing and representing geometric compositions, as well as many of the same sorts of techniques highlighted in the prior examples. Throughout the next three weeks, as students advanced their own analysis, we constantly made reference to these historical examples.

Leading up to our midterm, the project was composed of two phases. First, students developed their own methods for describing and representing the geometric principles of specific features of their organism. One student, for example, analyzed the curvature of a manta ray’s fins (figure 15), reconstructing their complex curvatures with a method using tangent lines and segments of perfect circles. Other students studied things such as the explicit and implicit curvatures of a honey bee’s head, or the geometries and implied planes of a snow crab’s legs and carapace (figure 16). Based on this
analysis, we then had students fold the methods of geometric description and representation they developed into a rigorous and systematic method for generating a new object (figures 17-18). We referred to this object as an “analogue” (figure 19), and it was intended as some sort of spatial and structural prototype which, through the methods they had developed to generate it, would embody particular formal properties, qualities, and characteristics of the organism they studied.

Figure 16. Analysis of snow crab carapace by Chloe Onbargi.

Figure 17. Translation of analysis of snow crab carapace into generative process by Chloe Onbargi.
Following the midterm, we provided students with a loosely defined architectural program—to design some structure for viewing an aspect of the landscape—and we asked students to select a landscape anywhere in the world in which to set their project. Proposals included structures for viewing the stars in the Negev Desert, experiencing the horizon from the cliffs of southern France, or weathering the 20’ changes in water levels of an Alaskan tidal flat as a sort of environmental pilgrimage (figure 20), to sitting on a lake and appreciating the reflected landscape in the hills of Bulgaria (figure 21), to feeling the wind and watching the sea on a Caribbean island, and on. In transforming their analogue, their structural prototype, we asked students to deploy the same methods of geometric description they’d
used before, but now to manipulate the underlying parameters of their designs to adapt their proposals to their specific context and use (figure 22).

In many cases, the language we used and ways we discussed projects with students could be described as “proto-parametric.” Many projects were, in a way, manually scripted, or procedures were used that loosely resemble computational processes. This can be best illustrated in the case of two particular projects: one based on the deer fern and the other on the skeleton of the opuntia cactus.
Figure 21. Project based on cannonball jellyfish by Galena Sardemova.

Figure 22. Project based on praying mantis by Linda Zeng.
In the case of the deer fern, the student studied the geometry, scalar change, and arraying of the leaf’s pineals, its leaflets (figure 23). In the analogue, these were adapted to develop a method for producing fern-like structures, where variables of scaling, folding, rotating, and arraying were manipulated in the pineals (figure 24). And then these fern-like structures were arrayed to produce surfaces. The project was then sited and readapted to an American pastoral landscape (figure 25). In essence, this project mimics the sorts of operations one might deploy with grasshopper, but here manually manipulating a collection of geometric and organizational parameters—like the scaling, folding, rotating, and arraying of elements.

Figure 23. Analysis of Deer Fern by Matt Fuchs.
Figure 24. Analogue of Deer Fern by Matt Fuchs.

Figure 25. Analogue of Deer Fern by Matt Fuchs.
Studying the skeletal structure of the opuntia cactus, another student developed a method for producing a cactus-like geometric composition with a manually scripted procedure. It began by identifying three sets of underlying grid lines to define a hexagonal structural grid, the dimensions, intervals, and angles of which could be subtly manipulated (figure 27). This grid was then explored regarding its potential to be translated into a three-dimensional structure, once more identifying parameters which could produce variation (figure 28). The scheme was then sited in the Negev desert as a structure for shade and stargazing (figure 29-30). Once again, parameters were manipulated in both in relation to a gently sloping site and the requirements for the structure’s inhabitation.

In addition to equipping students with a variety of foundational skills and conceptual tools for producing architecture, with this sequence of studio exercises we’ve also aimed to teach students ways of thinking that prepare them for our current post-digital era. We’ve tried to culture their abilities to think analytically and abstractly. We’ve also sought to help them, with reference to historical precedents in geometric description, to develop a rigorous and systematic approach to the analysis and design of complex form through the control of its underlying geometries. And finally, we’ve tried to instill in them a flexible way of thinking about the mediation between digital processes and their adaptation to questions of context and use.
Figure 27. Analysis of cactus skeleton structure by Dillon Horowitz.
Figure 28. Analogue of cactus skeleton by Dillon Horowitz.
Figure 29. Project based on cactus skeleton by Dillon Horowitz.
Figure 30. Project based on cactus skeleton by Dillon Horowitz.