A Competence Model for Handwriting

John Hollerbach

It is proposed that handwriting production is fundamentally an oscillatory process arising from two orthogonal joints. Letter shapes emerge from an oscillation train by a process of constrained modulation. The choice of underlying oscillation and modulations limits the diversity of letter shapes and gives rise to a common writing style. The model was tested by synthetic production of human-like cursive script with a mechanical arm under computer control. In this simulation the vertical joint acts as the driving joint, the horizontal joint as the shaping joint. Various force constraints on the oscillations and modulations are proposed.

One approach towards understanding handwriting is to attempt to produce handwriting artificially with a mechanical device. Artificial production serves to test models of handwriting and to hit upon insights into the handwriting process not readily obtained by measurements on humans.

In the present study a six-degree-of-freedom computer-controlled mechanical arm — the VICARM of the MIT Artificial Intelligence Laboratory — was used to synthesize handwriting (Figure 1). The VICARM is a commercially available mechanical arm designed by Victor Scheinman while at MIT. A rapid ballistic movement comparable in speed to human speeds can be produced by controlling currents to torque motors at each joint. Position and velocity sensors at each joint allow the computer to monitor the movement, although this feedback was not used to correct the open loop handwriting trajectories. Other work on simulation has used primarily graphics displays to generate handwriting according to a computational model of the handwriting process (McDonald, 1966; Yasuhara, 1975); Koster and Vredenbregt used a mechanical device similar to a chart recorder.

The emphasis in my research has been to determine if there is an underlying control strategy in the production of diverse letter shapes. One alternative explanation is that each letter has an iso-

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Author's address: Psychology Department, MIT, Cambridge, MA 02139.
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Figure 1. The VICARM, a six-degree of freedom manipulator used in the mechanical writing.

lated motor program — idiosyncratically formed by each individual — that is combined with other letters by a general linking process. Is handwriting a sequence of isolated letters strung together, or do superficially different letters share common production features? Eden (1962) developed a stroke vocabulary by decomposing letters into sequences of straight strokes and corner strokes; while different letters are related in this scheme by sharing strokes from the vocabulary, there is some question as to whether his decomposition is a good model for how people form letters or whether it is a formal artifact. Other past research, while not necessarily adopting the position that letters have isolated motor programs, has focused for the most part on duplicating human written letters or words as precisely as possible without concern for underlying meaning to the parameter values to their equations (Mermelstein, 1964; McDonald, 1966; Yasuhara, 1975).

It will be proposed that handwriting be viewed as the constrained modulation of an underlying oscillation pattern. Letter shapes are superimposed on the oscillation pattern and are subjected to the operation of the oscillation as a whole. Seemingly different letter shapes have the underlying oscillation pattern in common, as well as the same constraints on force patterns. The rest of this paper explores constraints to the oscillation pattern and their implications for producing letter shapes.

There is precedence for postulating oscillatory mechanisms in locomotion research (Grillner, 1975). More subjectively each of us

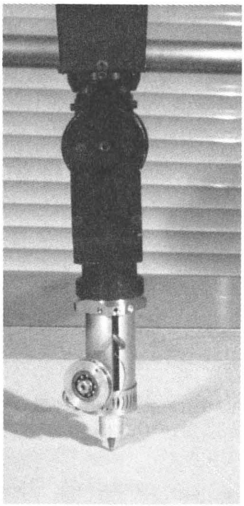


Figure 2. A spring loaded cartridge attachment to the VICARM keeps the pen on the writing surface.

can feel a steady rhythm in fast fluent handwriting that varies little from letter to letter. There is also a simplicity of control of oscillating mechanisms which may literally be responsible for letting us write and think at the same time.

This model of handwriting rests more on adequacy and simplicity criteria than it does on experimental evidence, mainly because the measurement apparatuses in past experimental work have not provided accurate enough measurements of position, velocity, and acceleration during writing. Parallel to this mechanical arm project I have designed and built an apparatus for more precise measurements; this research — that would allow assessment of the model against human performance — is still in progress. Therefore, this model of handwriting should be viewed as a competence model rather than as a model that performs exactly as humans do. The model represents a simple motor control strategy that has few parameters (and hence is testable) and that captures some of the complexity of handwriting.

The only letter shapes dealt with here are lower-case cursive script. No claim is made about the formation of capital letters, about dotting *i's*, or crossing *t's* and *x's*. Nor do we consider gross movements of the hand and arm between words or between lines of script. The problems of pen grasping and pen-paper contact have also been finessed in this research by design of a special spring-loaded pen attachment to the VICARM (Figure 2).

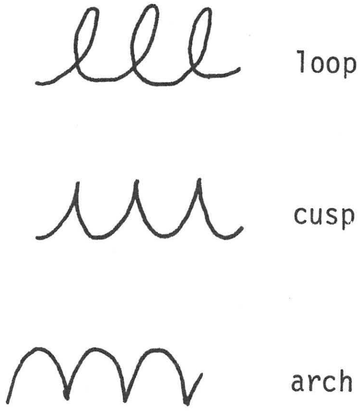


Figure 3. The three basic top corner types in handwriting.

Within these limitations the production of handwriting has been ascribed to two joints, a vertical joint and a horizontal joint in the plane of the paper (Denier van der Gon and Thuring, 1965). For human writing these joint roles are normally fulfilled by fingers and wrist, respectively. The best joints in the VICARM for this purpose are the wrist joint for vertical movement and the shoulder joint for horizontal movement; the disparate size of the joints was found unimportant for the production of realistic writing.

An Underlying Oscillation

The underlying oscillation pattern in the VICARM writing is fundamentally a down-up movement traveling from left to right. The vertical joint plays the role of oscillator: it drives the movement with rhythmic down-up movements. The horizontal joint acts as the shaping joint: by carrying the writing from left to right at different speeds and at different points in the vertical oscillation, it produces letter shapes.

Over a background of repetitive down-up motion, the horizontal joint shapes the top and bottom turning points into various types of corners suitable for particular letter shapes. There are three basic corner types: loop, cusp, and arch (Figure 3). Unless expressly stated otherwise, all samples of writing in the figures were produced by the VICARM. All letters can be decomposed into a sequence of these corners. Within each corner type there is considerable leeway in shaping: loops can be large or small, arches can be rounded or angular. An "individualistic" writing style can be obtained by a consistent interpretation of corner shape.

As an example of the letter formation process, Figure 4 indicates the transition between an oscillation pattern and a VICARM rendering of *vial*. The oscillation pattern couples repetitive down-up movements in the vertical joint with rightward start-stop movements in the horizontal joint. Different horizontal patterns at the bottom of the vertical oscillation would give a different shape to the underlying oscillation; this figure represents just one example of such a pattern. In (b) the vertical joint modulates force amplitude for letter height; the second down-up movement has been shortened for the connection between *v* nad *i*, and the sixth up-down movement has lengthened for the letter *l*. In changing letter height in an oscillation train, one has the choice of either modulating the amplitude or the frequency; in the present model the formation of tall letters is assumed to occur by a process of amplitude modulation without changing frequency. In (c) the horizontal joint overlays a rightward corner pattern at the bottom of the *a* and a leftward corner pattern at the top of the *l* to produce the *vial*. Although letter height and top and bottom corner shape adjustment were accomplished at different times in this example, they are not necessarily independent processes.

Force Constraints

A motor program corresponding to a particular corner type is a single acceleration-deceleration pattern. Loops come from leftward accelerating patterns, arches from rightward accelerating patterns, and cusps from patterns that decelerate to a halt exactly at top or bottom turning points. In the following, positive accelerations will be considered upward for the vertical joint and rightward for the horizontal joint.

Within this framework there is still considerable leeway in forming corner patterns and in setting up the vertical oscillation. A number of force constraints have been applied that limit the number of possible corner patterns and oscillations but preserve the ability to produce a sufficiently diverse set of letter shapes. The first two constraints restrict the vertical oscillation patterns; the last three contraints restrict both horizontal and vertical patterns. As mentioned earlier, these constraints are derived mainly from simplicity criteria.

1 *Strict rhythmicity.* Cycles of an oscillation always have the same duration. Thus tall letters are written as fast as short letters.

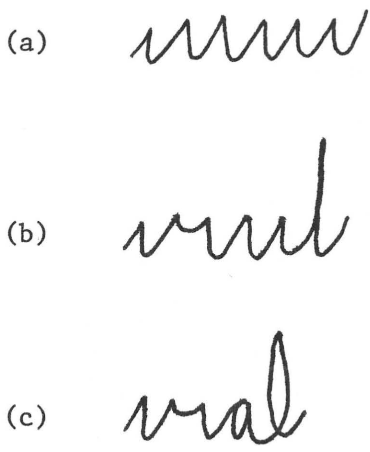


Figure 4. Conceptual process of letter formation beginning with the oscillation train (a), and ending with the cursive rendering of *vial* in (c). In (b) the vertical joint modulates for letter height. In (c) the horizontal joint applies various corner patterns at appropriate points in the oscillation.

2 *Symmetric oscillation.* Downstrokes and upstrokes of the same height have the same duration; within a downstroke or an upstroke the acceleration and deceleration periods have the same duration.

3 *Constant force tendency.* The force amplitude stays at a constant plateau during an acceleration or deceleration period.

4 *A fixed time quantum.* Time is quantized into a series of fixed steps. During a time step, force is held constant; at the end of one step and the beginning of the next the force may change amplitude. The time quantum stays the same for all writing speeds and sizes.

5 *Synchronous joint activation.* The time quanta have the same size for different joints. The beginnings and ends of the time quanta occur at the same time.

To illustrate the effects of these constraints on the motor patterns, the force patterns for two cycles of the writing of Figure 4 are diagrammed in Figure 5. Focusing first on the bottom left plot, which represents the vertical force pattern for two cycles of the top writing in Figure 4, the abscissa has been divided by the ticks into single quantum units. Changes in force amplitude are seen to occur only at integral units of the time quantum, satisfying constraint 4. The time quanta for the horizontal and vertical joints have the same duration and their onsets are synchronized, satisfying constraint 5.

Following constraint 1, the two cycles represented in Figure 5 have the same duration, namely four time quanta; this oscillation pattern produces the fastest possible writing under the constraints. Each quantum acts to accelerate or decelerate the pen in a given direction. The net amount of acceleration must of course be balanced by an equal amount of deceleration, assuming no losses in the system. For example, the vertical quantum 1 causes a downward acceleration from the top corner. Quantum 2 decelerates the pen exactly to a halt at the bottom corner. Quantum 3 then accelerates the pen upwards, and quantum 4 decelerates the pen to an exact halt at the top corner. This process continues for each cycle of the oscillation.

According to the symmetric oscillation constraint 2, the downstrokes are required to take the same amount of time as the upstrokes; in this example each such stroke requires two time quanta. Moreover, the acceleration and deceleration periods in the downstrokes and upstrokes are of equal duration: quantum 1 matches quantum 2 for the downstroke, quantum 3 matches quantum 4 for the upstroke. Constraints 1 and 2 are trivially met in this example, but for longer cycled oscillations there are more possibilities. Constraint 3 requires that the two quanta 5 and 6 in the rightward acceleration of the horizontal joint have the same force amplitude.

In the bottom right diagram in Figure 5 the vertical force profile for the two down-up cycles going into the production of the *l* show different amplitudes between deceleration from the downstroke to acceleration for the upstroke. Since the constraints prevent duration scaling, force amplitude scaling is required to produce the taller letter *l*.

Slower oscillation patterns allow the horizontal actuator more time to shape letters, in particular more time to incorporate leftward movements. With an 8-quantum cycle, a garland chain with leftward movement and more rounded bottoms can be obtained (Figure 6). When modified, the garland chain becomes the *vial* below. A feature of this modification is hesitation at sharp corners such as the *i*, where the top loop of the garland is aborted. The hesitation is required to keep the writing in phase with the oscillation.

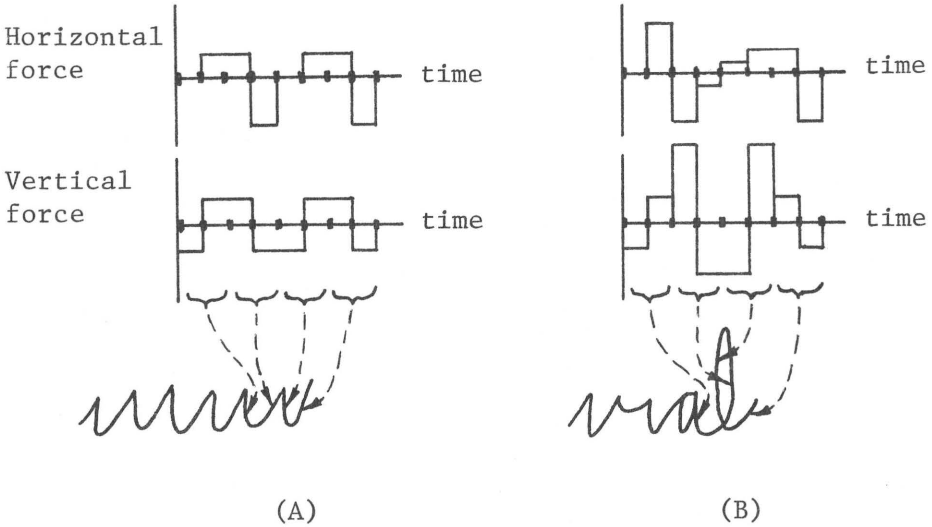


Figure 5. (A) Horizontal and vertical force patterns by the actuators produce the smoothed sawtooth below. Two cycles of the oscillation are shown in the force patterns. The correspondence of downstrokes and upstrokes with portions of the vertical force pattern are indicated by dashed lines. The ticks represent single time quantum units. (B) Modulation of the force pattern in (A) produces *vial;* the two cycles forming the *l* are shown in the force diagram.

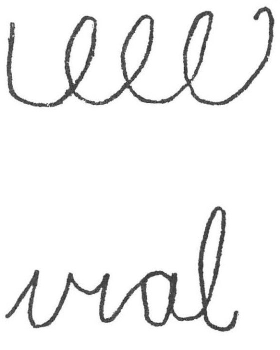


Figure 6. An eight-quantum cycle allows the production of a rounded pattern of *e's*. Modulation of this pattern yields the *vial* rendering below.

Horizontal Motor Programs

The horizontal joint, which does not oscillate as does the vertical joint, may have different amplitudes and durations in adjacent acceleration-deceleration periods. The only constraint is that the negative deceleration brings the movement exactly to a halt. Additionally there can be a quantum of coasting or zero acceleration between positive and negative accelerations for reasons of shape production. Thus all corner patterns are either of the form $a^m b^n$ or $a^m 0 b^n$; where a^m means m quanta of positive acceleration amplitude a, and b^n means n quanta of negative acceleration amplitude b, subject to the constraint ma = nb.

Once a horizontal corner pattern has been selected, its onset must be synchronized relative to the vertical joint movement. This synchronization is specified with respect to the vertical top or bottom turning point. Depending on how one shifts, or overlays, this pattern relative to the turning point, different corner shapes are obtained. In practice there are two overlays that produce the most generally useful shapes: (1) the acceleration ends precisely at the turning point; (2) the acceleration ends one quantum after the turning point. In addition useful shapes are obtained only if the acceleration is about the same length as the deceleration; otherwise strongly asymmetric shapes result. This restriction may be expressed as limiting the difference between acceleration and deceleration durations to no more than one quantum.

The important features of the horizontal patterns can be summarized in terms of a simple motor program vocabulary. The elements of this vocabulary that go into the making of a "word," or motor program, are presented in Table I.

A "word" in this vocabulary thus has three "letters"; for example a 7SD word represents a pattern with a total of seven time quanta symmetrically arranged about a coast quantum (i.e., an $a^3 0 b^3$ pattern) overlayed so that the last acceleration quantum a ends at the vertical corner. The existence of a coast quantum can be deduced from 1 and 2; for example, a 4L corner pattern has a coast quantum, a 5L pattern does not. Table II gives some examples of horizontal corner patterns and a rough description of the shapes produced. The corner patterns are diagrammed in Figure 7 and their shapes appear as bottom corners in Figure 8.

An association library can be formed that contains horizontal corner patterns formed according to the vocabulary of Table II and

Table I. Corner Shape Vocabulary

1	Total number of time quanta			
2	Symmetry	S	symmetric	$a^n(-a)^n$ $a^n0(-a)^n$
		L	asymmetric left	$a^{n+1}b^n$ $a^{n+1}0b^n$
		R	assymmetric right	a^nb^{n+1} a^n0b^{n+1}
3	Overlay	A	accelerating through corner	
		D	acceleration stops at corner	

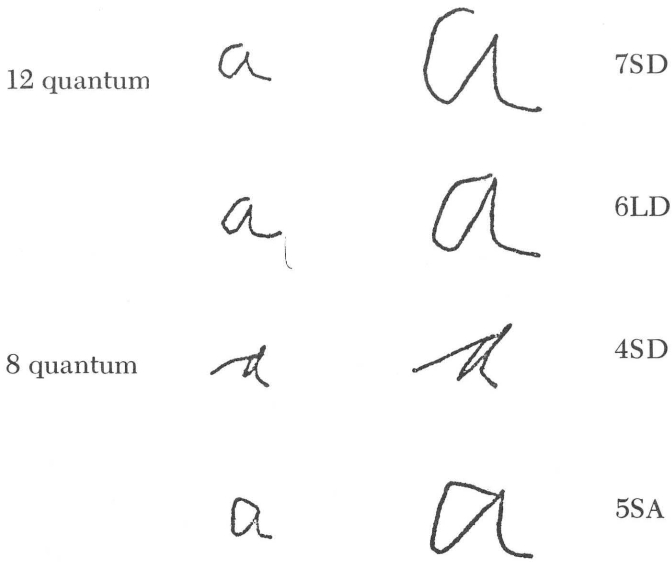
a qualitative shape description corresponding to each pattern. Each horizontal corner pattern is a ready to use motor program that needs only the amplitude values for instantiation. To produce a desired letter shape, the letter is broken down into a sequence of qualitative corner shapes and the appropriate horizontal motor programs can be retrieved from the library. The breakdown of letter shapes themselves can be kept in a separate table.

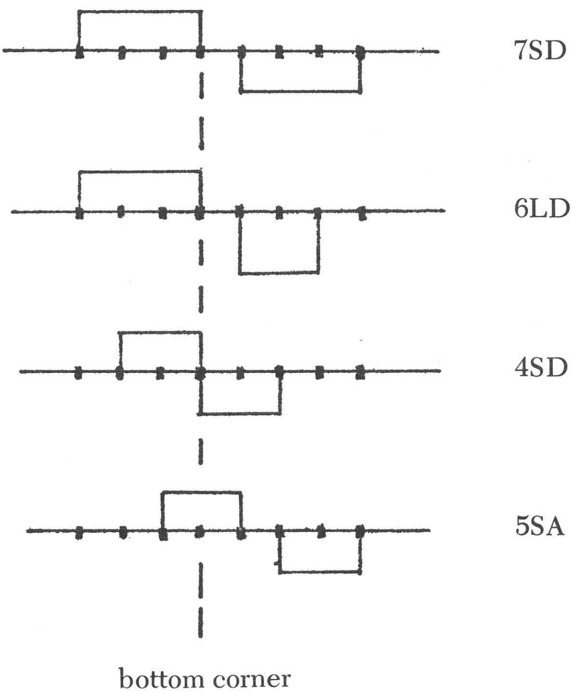
This restricted repertoire of corner patterns is adequate to capture many of the shape variations seen in human handwriting. As an illustration an attempt was made to duplicate four *a's* as drawn by different human subjects, taken from Koster and Vredenbregt (1971). These *a's* appear paired with their VICARM facsimilies in Figure 8, the former the smaller writing on the left and the latter the larger writing on the right. The *a's* as represented in the figure are actual size; the larger writing is favored for the VICARM for mechanical reasons. Appearing to the right of each VICARM letter is the horizontal pattern for the bottom corner in the language of Table II. The output current to the horizontal torque motor corresponding to each of these patterns is diagrammed in Figure 7.

As is evident from the figure the duplication is fairly good, especially considering the differences in mechanics. The process

Table II

Rounded	7SD
Sharp cornered	4SD
V shaped	5SA


12 quantum
7SD
6LD
8 quantum
4SD
5SA


7SD
6LD
4SD
5SA
bottom corner

of manufacturing these shapes was accomplished first by choosing an appropriate oscillation speed and then selecting appropriate corner patterns. The first two *a's* are rather rounded, and a relatively long twelve-quantum cycle was required for correct shaping. The last two *a's* are more angular, calling for a faster eight-quantum cycle.

The Inherent Simplicity of Handwriting

Though human handwriting may be executed under a different set of strategies from the VICARM writing, the VICARM results suggest that handwriting is an inherently simple task. The simplicity arises because superficially different letters share in the VICARM writing common production features. Once an oscillation is set up under the various constraints, a limited number of corner types suffices to produce most letter shapes. In a sense these strategies may be viewed as a trick to minimize the complexity of handwriting.

As a rough numerical estimate to indicate the complexity of the VICARM writing, the number of required corner types is in the vicinity of twenty. The number of force levels found necessary in the VICARM writing to cover acceptably the variety of letter shapes is also small because casual observation shows writing to fall naturally into letters of three or four different heights and perhaps ten different widths. The implication of this small number is that there need be only limited control in handwriting in shaping letters.

Of course, the more exact a duplication of particular corner shapes is required, the larger must be the number of different corner types. This number also does not take into account personal idiosyncracies such as peculiar letter shapes, combined printing and writing, and drastic changes in the style of writing. By writing slowly, it is possible to produce a much greater variety of letter shapes than by writing fast; it is not clear, however, that fast and slow writing are produced by humans in the same way. Nevertheless, for a regular style of writing such as the VICARM's, relatively few corner types suffice to produce acceptable writing.

Figure 7. Diagram of force patterns by the horizontal actuator corresponding to Table II and to the bottom corner patterns of Figure 8.

Figure 8. Four *a's* from different human subjects (Koster and Vredenbregt, 1971) on the left are paired with their VICARM fascimiles on the right. The top two *a's* were produced with a twelve-quantum cycle, the bottom two *a's* with an eight-quantum cycle.

We have only touched on earlier the ideation process involved in converting the mental image of a word into a sequence of handwriting strokes. It was suggested that letters are decomposed into a sequence of qualitative corner shapes that can index into an association library of motor programs. The connections between letters are also just different kinds of strokes and corner shapes, and are treated similarly. It may be the case that context effects interfere in this process; for example, by an unlikely juxtaposition of letters such as jz. The real issue here is what the elementary unit in handwriting is: is it the letter, the syllable, or the word? An unusual combination such as jz may cause difficulty in writing because it is not an elementary unit or a composition of elementary units. The resolution of this issue is beyond the scope of the present work.

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