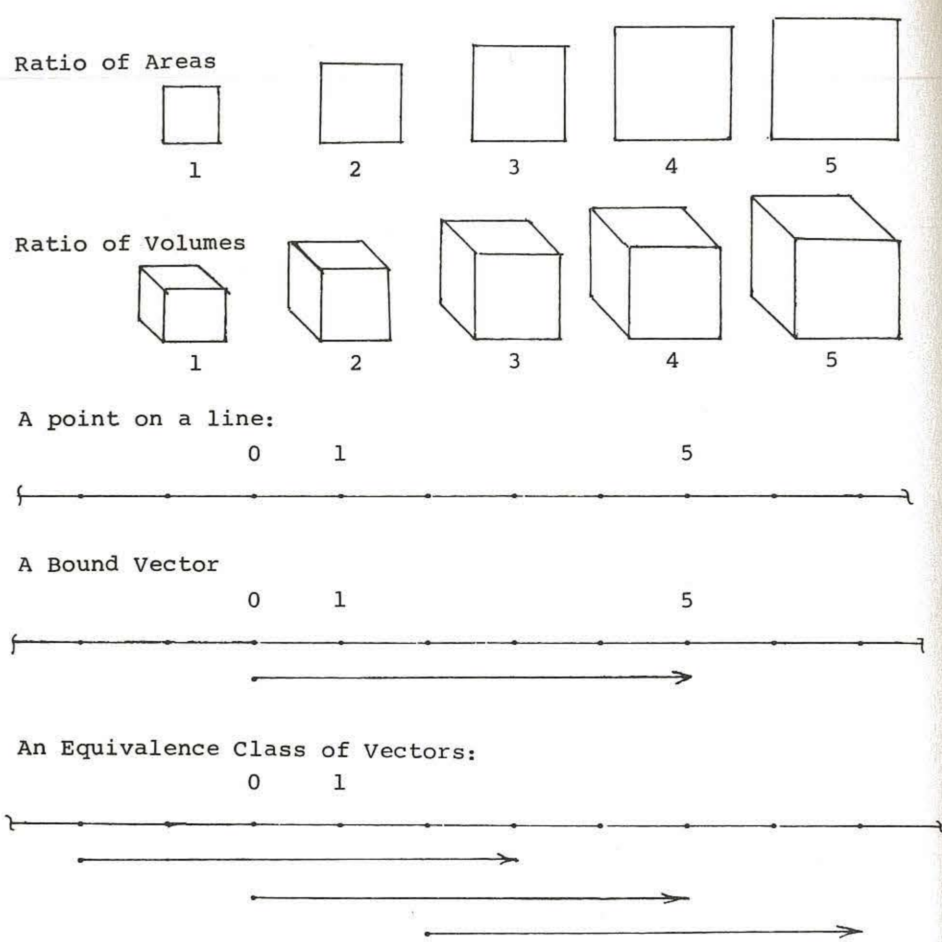

An equivalence class of vectors can also correspond to an increase of n, as represented by the operator + n.

Language Acquisition through Mathematical Symbolism

Francis Lowenthal

We noticed that the use of a non-verbal formalism can favour cognitive development (in the frame of the elementary school) in problem children as well as in normal children. An example is given to show how a formalism inspired by mathematics can be used to aid the development of the verbal language of 8- to 9-year-olds. We will then analyze the results and try to discover the cause of success we observed.

First we must specify which symbolic systems and which mathematical formalisms to use. In a previous paper (1980a) we stated, "We think that the main factor of cognitive development is *manipulation of representations*." In another paper (1980b) we claimed that any representation system which satisfies the six following criteria can be used: the system must be *non-ambiguous, simple and easy to handle, non-verbal* (to avoid conflicts with the developing verbal language); it must also be *supple enough* to enable the child to become conscious of what he knows but cannot verbally express; it seems essential that such a system should be *suggestive of a logic* and could be introduced and used *in the frame of games* (to enable us to use it easily with young children).

We wanted each of our systems to be suggestive of a logic; this is why we decided to choose representation systems used in mathematics. This requirement enabled us to represent our symbolic system in terms of a game. The rules of a game are explained and the children must collectively build a representation. This is the first stage of their work: *the synthesis*. They must then modify the representation and only respect technical constraints while doing so. They then reach the last stage: the analysis of the new representation and the collective discovery of the rules of the new game. Similar exercises can be invented for language acquisition.

What follows is a report of an actual lesson during which we asked the children "to tell a coherent story corresponding to a given representation." We will thus describe the adventures of a class of normal 8- to 9-year olds. The representation system we chose is that which is used in the new math (Papy 1968). Objects are represented by

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dots and relationships between objects by multicoloured arrows. Each dot represents exactly one object (which can have several names) and each colour represents exactly one relationship; 2 dots are associated to 2 different objects and 2 colours to 2 differing relationships. The children suggested the starting diagram (Figure 1). They decided to use only two kinds of "arrow-relationships": red and green ones. (For technical reasons we will represent red by a discontinuous line and green by a continuous line.)

First stage

Ronald produces the diagram shown in Figure 1, but does not say anything. Rudy asks immediately: "Does one split Magali into two?" but Ronald does not answer. Fabrice notices: "The dot below has no name," and Rudy tries to explain: "The green arrow says, 'to go to the park,' so Magali goes to Nicolas' and Nicolas goes and sleeps in the park."

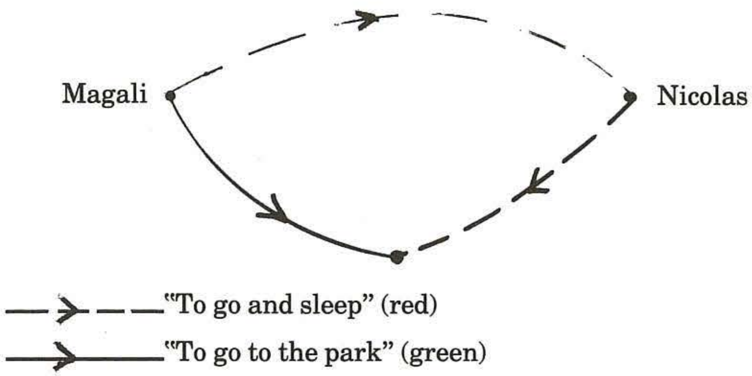

Figure 1.

Second stage

Isabelle suggests calling the third dot "Marie" and the whole class accepts this. Rudy, who is still thinking in terms of games, says: "One game, it will be the park; the other one, it will be "to sleep." We should add more arrows." (He probably assumes that there are two "games" for Magali).

Fabrice asks: "Does Magali go to the park at Marie's?" He adds: "Magali goes and sleeps at Nicolas'. Nicolas goes and sleeps at Marie's. Magali goes to the park at Marie's." Pascal corrects him: "... with Marie."

The teacher interferes then and asks: "Fabrice made a mistake. Why?" Isabelle suggests: "Magali goes to the park at Marie's." "At Marie's?" asks the teacher. The children do not seem to find improper the word "at." The teacher insists then upon the relationship "to go and sleep." Catherine re-reads the picture: "Magali goes and sleeps at Nicolas'." Fabrice suggests: "Magali goes and sleeps Nicolas" but Catherine proposes: "'Magali, go and sleep!' says Nicolas"; and Pascal: "Magali goes and sleeps with Nicolas."

"'At' or 'with'" asks the teacher, while Rudy wonders: "May we change the arrows?" Fabrice wants to add a little word to "to go and sleep" and obtains: "to go and sleep *at* [. . . 's house]." The class thus obtains a text which is read by Silvie: "Magali goes and sleeps at Nicolas'. Nicolas goes and sleeps at Marie's."

"The green arrow annoys me," announces Rudy. Bertrand notices: "It gives: Magali goes to the park Marie." "To go to the park *of* Marie" says Rudy, but this is rejected by the rest of the class, while Catherine suggests adding "with" at the dot called "Marie." The text becomes then: "Magali goes and sleeps at Nicolas'. Nicolas goes and sleeps at Marie's. Magali goes to the park with Marie."

Third stage

Pascal notices that "If Magali goes and sleeps, she cannot go into the park." Fabrice puts both actions in a time perspective: "Magali goes and sleeps at Nicolas'. Tomorrow Nicolas will go and sleep at Marie's. The day after tomorrow Magali will go to the park with Marie."

The teacher, thinking of the symmetry implied by the word "with," asks: "There is a problem in this story. Which one?" He has the impression that the pupils feel that there is a qualitative difference between the two actions of Magali, but that they cannot express verbally and correctly the idea.

"If Magali goes to the park with Marie . . . " starts the teacher, and Fabrice continues: "Then Marie goes to the park with Magali." The pupils then suggest adding an arrow and obtain the diagram shown in Figure 2. Catherine notices: "Both are going," and the teacher adds: "To go somewhere with somebody; the persons are together."

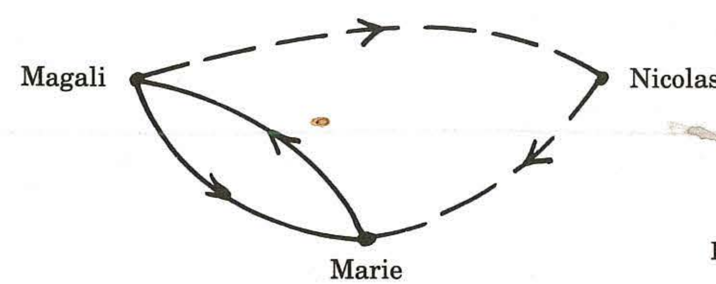

Figure 2.

Sylvie finally tells the complete story: "Magali goes and sleeps at Nicolas'. Nicolas goes and sleeps at Marie's. Magali goes to the park with Marie. Marie goes to the park with Magali." The whole class gives these data the shape of a story: "Magali goes and sleeps at Nicolas'. The next day Nicolas will go and sleep at Marie's. The day after tomorrow, Magali and Marie will go together to the park." Catherine uses the time variable to create another story, using the same basic data: "Yesterday Magali went and slept at Nicolas'. Yesterday Nicolas went and slept at Marie's. To-day, Magali and Marie will go to the park together."

During this lesson the children successfully and graphically produced a situation. Starting from this situation, they created a story. Their teacher can now use the story *created by the children themselves* to introduce, in the frame of the language course, exercises about conjugation, about personal or relative pronouns, or even subordinates.

Analysis of this example

At the first stage the children create a simple situation: there are objects and relationships. We are at the object-language level, not at the metalanguage level. There is not really a story. At the second stage we notice that the diagram suggests (to the children) complements which are required from a logical point of view (naming of all the dots, correct statement of all the relationships). There are already sentences, but not yet a story.

During the third stage the children correct apparent contradictions, thanks to the explicit introduction of variables which remain implicit in an ordinary conversation. This concerns the properties of certain words (e. g., symmetry for "with") but mainly the introduction of the time variable. The children are, nearly constantly, the initiators of the action, not the teacher. The children choose freely the conventions they want to use. The classroom dynamics plays an important role: one child proposes an idea, the whole class criticizes it, and after discussion all accept it — or reject it.

Discussion

One diagram is used as starter. During their discussion the children modify many things. They first try to adapt the story to the diagram, then the diagram to the new version of the story. It seems important to notice this back-and-forth process during which, in this case, time is introduced. More generally comments about the representation and

assessments of the value of one diagram compared to another appear in the children's speech.

Object-language is that part of language concerning descriptions of objects, or relations between actual objects, while metalanguage is that part of language concerning what is said about these descriptions or relations. "My pencil is broken" belongs to object-language, but "'My pencil is broken' is a correct sentence" belongs to metalanguage. When we use the technique described in this paper, we pass easily, especially in the way time is introduced, to the level of metalanguage and the children eventually create a coherent story. We think that our symbolic representations are useful, mostly because they enable the children to distinguish clearly between object-language (associated to the representations) and metalanguage (what is said about the representations). We think that this concrete distinction between object-language and metalanguage might be the factor which favours the children's cognitive development. A young child is able to use representations, but not always to state or to notice that two different representations can be used for the same object, for the same story. This is a problem of non-identity or non-conservation. But this is no longer true if we ask the child to compare concrete representations and to *say what he notices* while doing so: the child will then rapidly learn what Piaget said this young child cannot learn.

Remarks

The representation system we used enabled the children to create a diagram. This diagram was only used as starter for the narrative process. The story the children created is wider than the diagram's frame. Moreover, although it is true that the construction of the story is based upon the diagram (the basic elements of the story are suggested by the diagram), it is nevertheless wrong to believe that the diagram is used to communicate a complete message. There are conventions established by the pupils, many things are implicit and are not mentioned; other problems are never solved. It is true that Magali goes and sleep at *and with* Nicolas (story told by Pascal and Sylvie)? Or does Nicolas go away and leave Magali alone (story told by Catherine)?

Mistakes to avoid

The pupils should build their story by themselves; the teacher should only guide them when needed, as little as possible. Our technique should certainly not be used too formally; it would block the pupils' activity. We must accept, for instance, the quasi-identification of "Nicolas" and "at Nicolas' house," accept that the pupils change the name of the third dot and call it "with Marie" instead of keeping "Marie," and explicitly add the "with" to the green relationship. We may never forget that it is difficult to tell a story corresponding to the starting diagram. The teacher must be supple and let the children modify the diagram when they want to do it and exactly as they want to do it. They must have the possibility to adapt the diagram to the story they wish to tell, and the story to the diagram they want to keep, in such a way that they slowly reach a solution which satisfies them. If they succeed in building a coherent story — but a story which does not even look like the starter — the teacher must be able to accept it: the terminology, the convention, the game's control belong to the children.

One must, at all cost, avoid dogmatic use of the technique, for dogmatism kills the children's freedom of expression. We must use representation systems which, thanks to inner technical constraints, suggest to the child the use of a logic which the teacher has hidden in it.

Conclusion

A non-verbal auxiliary formalism can serve as guide to the child's thought. If this formalism, or representation system, is used in a non-dogmatic way, it enables the children to build a coherent story through successive adaptations that they suggest. This story can be graphically represented by the proposed formalism. In this case one should use the definitions formulated by some children and accepted by the whole class. Such a formalism is also useful because the teacher, when choosing the symbols and imposing upon them the technical constraints, can hide a logic in the system. The teacher can thus choose a logic which the children will use nearly spontaneously. Moreover, such a formalism enables the teacher to visualize the difference between object-language and metalanguage.

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Communicating Mathematics: Surface Structures and Deep Structures

Richard R. Skemp

A distinction is made between the surface structures (syntax) of mathematical symbol-systems and the deep structures (semantics) of mathematical schemas. The meaning of a mathematical communication lies in the deep structures — the mathematical ideas themselves, and their relationships. But this meaning can only be transmitted and received indirectly, via the surface structures; correspondence between deep and surface structures is only partial. Some resulting problems of communicating mathematics are discussed, and some remedies suggested.

The power of mathematics in enabling us to understand, predict, and sometimes to control events in the physical world lies in its conceptual structures — in everyday language, its organised networks of ideas. These ideas are purely mental objects: invisible, inaudible, and not easily accessible even to their possessor. Before we can communicate them, ideas must become attached to symbols. These have a dual status. Symbols are mental objects, about which and with which we can think. But they can also be physical objects — marks on paper, sounds — which can be seen or heard. These serve both as labels and as handles for communicating the concepts with which they are associated. Symbols are an interface between the inner world of our thoughts, and the outer, physical world.

These symbols do not exist in isolation from each other. They have an organisation of their own, by virtue of which they become more than a set of separate symbols. They form a symbol system. A symbol system consists of

a set of symbols	corresponding to	a set of concepts

together with

a set of relations between the symbols	corresponding to	a set of relations between the concepts.

What we are trying to communicate are the conceptual structures. *How* we communicate these, or try to, is by writing or speaking symbols. The first are what is most important. These form the *deep structures* of mathematics. But only the second can be transmitted and

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