Before Numerals

Denise Schmandt-Besserat

The paper deals with the development of counting devices in the ancient Middle East between 10,000-3000 B.C. and, in particular, bone tallies, clay tokens, and numerical notations on clay tablets. These technologies handled plurality in increasingly abstract terms. Data is tested against a model for the development of abstract numbers proposed by the historian of mathematics Tobias Danzig.

Introduction

In previous years I have studied the role played by prehistoric counters in the origin of writing.[1] I am presently studying the role played by the same counters in the origin of counting and, in particular, in the origin of abstract numbers.[2]

I first define some terms used in the discussion. *Numerals* are symbols to represent abstract numbers. Abstract means removed from the concrete reality. *Abstract counting* refers to using number concepts abstracted from any particular concrete entity. Our numbers 1, 2, 3, etc. . . . are expressing the concepts of oneness, twoness, threeness as abstract entities divorced from any particular collection. As a result 1, 2, 3 . . . are universally applicable. Concrete counting, on the other hand, does not abstract numbers from the things counted. As a result, in *concrete counting* the number words that express the concepts "one," "two," "three," etc., differ according to whether, for instance, men, canoes, or trees are being counted. These different sets of number words, which change according to the category of item counted, are called *concrete numbers.* Such examples as twins, triplets, and quadruplets to count children of a same birth is the closest analogy to concrete numbers in our own society. It is well understood, however, that in our society such special numerical terms which refer to particular groups are not really used for counting whereas concrete numbers were. *Counting in one-to-one correspondence* consists of matching the items to be counted by an identical number of counters. For instance, matching each sheep of a flock with a pebble. This method of counting does not require any concept of numbers.

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Author's address: Center for Middle Eastern Studies, University of Texas, Austin, TX 78712.

The Hypothesis

Danzig,[3] Smith,[4] Kramer,[5] and Flegg,[6] to name only a few, are among the historians of mathematics who have postulated that there were three main steps in the evolution of counting: (1) one-to-one correspondence, (2) concrete counting, and (3) abstract counting.

1 *One-to-one correspondence*

The historians of mathematics quoted above hypothesize that, ages ago, counting consisted only in the repeated addition of one unit with no idea of cumulative amounts. Tribes such as the Vedda of Ceylon never reached much beyond this level in historical times. They counted coconuts, for instance, by matching each coconut with a stick. For each stick added they counted "and one more" until the collection of coconuts was exhausted. Then, they merely pointed to the resulting pile of sticks saying "that many."[7] At this stage, in other words, people lacked concepts for numbers. Collections were conceived, therefore, as series of individual disconnected entities rather than as coherent wholes.

2 *Special numerations*

At this second stage the notion of sets is suggested to have been acquired. It would have fused, however, the concepts of number and of the objects counted. As a result, different things would have been counted with different numerical expressions or concrete numbers. This is inferred from languages where the words for numbers change according to the things counted. Menninger cites as an example the Fiji Islanders who call ten boats "bola" and ten coconuts "boro."[8] One of the most quoted example of concrete counting is that the British Columbia tribes studied by Franz Boas.[9] The concrete numbers they used to count men, canoes, long objects, flat objects, round objects or time; measures or other items are illustrated in Table I. Diakonoff recently published an article dealing with concrete numbers in which he gives the example of Gilyak, a language spoken on the River Amur, which had no fewer than twenty-four classes of numbers. For instance, the word used to express the number "2" was *mex* when referring to spears and oars; *mik* for arrows, bullets, berries, teeth, fists; *meqr* for islands, mountains, houses, pillows; *merax* for eyes, hands, buckets, footprints; *min* for boots; *met* for boards and planks; *mir* for sledges, etc. . . .[10] There are numerous remnants of such usage among Paleo-European,[11] Paleo-Asiatic, Micronesian, and Afrasian languages[12] suggesting that a stage of special numerations for individual classes of

Number	Men	Canoes	Long Objects	Flat Objects	Round Objects	Measures	Counting
1	k'al	k'amaet	k'awutskan	gak	g'erel	k'al	gyak
2	t'epqadal	g'alpeeltk	gaopskan	t'epqat	goupel	gulbel	t'epqat
3	gulal	galtskantk	galtskan	guant	gutle	guleont	guant
4	tqalpqdal	tqalpqsk	tqaapskan	tqalpq	tqalpq	tqalpqalont	tqalpq
5	kcenecal	kctoonsk	k'etoentskan	kctonc	kctonc	kctonsilont	kctonc
6	k'aldal	k'altk	k'aoltskan	k'alt	k'alt	k'aldelont	k'alt
7	t'epqaldal	t'epqaltk	t'epqaltskan	t'epqalt	t'epqalt	t'epqaldelont	t'epqalt
8	yuktleadal	yuktaltk	ek'tlaedskan	yuktalt	yuktalt	yuktaldelont	guandalt
9	kctemacal	kctemack	kctemaetskan	kctemac	kctemac	kctemasilont	kctemac
10	kpal	gy'apsk	kpeetskan	gy'ap	kpeel	kpeont	gy'ap

Table I. The Tsimshians of British Columbia used these various number words according to whether they were counting men, canoes, long objects, flat objects, round objects or time; measures and any other item. The use of different numeration systems to count different items is called "concrete counting."

items may have preceded the acquisition of abstract numbers in several parts of the world. In the case of Gilyak where the different sets of numerals seem not totally unrelated, but constitute perhaps only modifications of the same root forms, it could be argued that the language inflected the numerical expressions according to the semantic categories they modified. The use of numerical classifiers in Japanese,[13] in Aztec and Maya languages[14] can be viewed, probably, also as relics of such concrete counting practice.

Certain English numerical expressions to express "two" and "many" are comparable to concrete numbers, for example, "a couple of days," "twins," "a brace of pheasants," "a pair of shoes," "a school of fish," "a flock of sheep," "a herd of cows," and "a pride of lions." These different words to express quantities in specific situations may suggest that in our own society there was a time when not only concrete counting was common[15] but when counting was restricted to concrete numerations limited to "one," "two," and "many."

What is our present knowledge on the evolution of counting in the ancient Middle East and, in particular, is there any evidence for the use of concrete counting? Diakonoff postulates that there is. The Soviet sumerologist and linguist argues that the many different numerical signs to express quantities, capacity, area measures, etc. . . . point toward an ancient tradition of concrete counting in proto- or prehistoric Mesopotamia.[16] As will be discussed below, the archaelogical material supports Diakonoff's hypothesis.

3 *Abstract numbers*

At this third and final stage, the concepts of numbers would have been abstracted from the items counted, giving rise to abstract numbers which could be applied universally, like our own concepts of one, two, three, etc. Smith remarked that in a number of societies, the words to express abstract numbers derived from a concrete numeration of particularly frequent use. He cites, for instance, the Niues of the Southern Pacific who counted with abstract numbers that meant literally "one fruit, two fruits, three fruits," whereas in other cases the words corresponding to our "one, two, three" were expressed by such words as "one grain, two grains, three grains," or "one stone, two stones, three stones."[17]

In sum, according to the hypothesis presented above, counting would have evolved in spurts followed by plateaus over an exceedingly long time. As Russell wrote, "It must have required many ages to discover that a brace of phaesants and a couple of days were both instances of the number 2."[18] The study on cognitive development in children carried out by Piaget recognizes comparable stages leading to the mastery of counting.[19] According to Piaget's analysis, children start by matching collections in a one-to-one correspondence at an early age, but associate quantities with numbers relatively late.[20] At present, no one can explain how this evolution takes place in the child except by a process of maturation.[21] We know equally little about the mind of early humans and, at present, do not understand the cognitive processes that led to the development of counting and its timing. It is to this issue that ancient reckoning devices and their proper interpretation may contribute new insights.

The evolution of reckoning technologies in the Ancient Middle East

Here we deal with artifacts found in excavations in the Middle East which have been identified as counting devices. These objects include tally sticks, tokens, and notations on clay envelopes and tablets. I analyze the way each device may have handled plurality and suggest that the archaeological data substantiates the hypothesis presented above in each of its three successive steps, as follows:

1 *Tallied bones used to count in a one-to-one correspondence*

Animal bones and antlers bearing series of notches found in Mesolithic sites of the Middle East about 10,000 B.C.[22,23] are the earliest artifacts interpreted by scholars as reckoning devices.[24] It is not surprising to find tallied bones as the earliest evidence for the art of counting in the ancient Middle East because notched sticks are among the most primitive reckoning devices that are attested from all parts of the world.

Whatever the Mesolithic notations represented, they seem to have functioned in a one-to-one correspondence. The markings appear to be case specific. That is to say, the same kind of notch would have stood, according to the occasion, either for a bison or a reindeer. Only the person, or persons, keeping tally could have known, therefore, what was being recorded.

Such notations would involve abstraction in the sense that one concrete object seems to be represented by one abstract notation. This would have had the effect of bringing together for scrutiny the repeated occurrences of the objects counted; however, there is nothing in the tallies that indicates any notion of sets. The notches are arranged in series of units which are apparently never articulated into quantified collections. The tallies seem to illustrate, therefore, the first level of counting, in a one-to-one correspondence.

2 *Tokens used for concrete counting*

I have presented evidence pointing to the use of tokens for counting between 8000-3100 B.C.[25] This token system seems to reflect the conceptual level at which only units of the same kind could be counted together.

The singularity of the token system lies in the multiplicity of shapes of the counters. Whereas the mesolithic tallies had employed a series of identical notches incised on a bone, tokens were now modeled in clay into specific, systematically repeatable shapes, easy to identify. While the series of notches could only be understood by those who had initiated them, a group of tokens could be identified at all times with units of a specific product. In other publications, I have argued that each shape stood for a unit of precise commodity.[26] For instance, a sphere seems to have equaled a large measure of grain and a cone a small measure of grain, whereas an ovoid probably represented a jar of oil (Figures 1 and 2).

I would like to emphasize here my assumption that the units of products expressed by the tokens should be understood as traditional containers in which the goods were dealt with in daily life. They would correspond to such measures of common usage such as "a pitcher of beer," "a carafe of wine," and "a mug of coffee." These units, in other words, should be considered as only casually standardized and entirely non-mathematical entities. Grain, for example, might have been handled in baskets of various usual sizes in which case the cone could stand for "a basket of grain" and the sphere for "a large basket of grain." The "basket" and the "large basket" would be used in different circumstances requiring different quantities of grain but the "large basket" would be in no way considered as a direct multiple of the "basket."

I have also argued that the tokens were used in a one-to-one correspondence. In other words, one jar of oil would have been represented by one ovoid, two jars by two ovoids, and so on.[27]

In spite of this one-to-one correspondence that characterizes the token system, it entails certain elements of abstraction. First of all, the units of real goods, such as quantities of grain and oil, are replaced by clay symbols, and this itself is the result of a process of abstraction. Secondly, the tokens abstracted the data from its context, thus allowing the accounts to abstractly manipulate goods. For example, the Sumerian accountant dealing with the administration of temple flocks using tokens did not need to take into account the actual whereabouts of the sheep involved.

On the other hand, the tokens remained concrete in several ways:

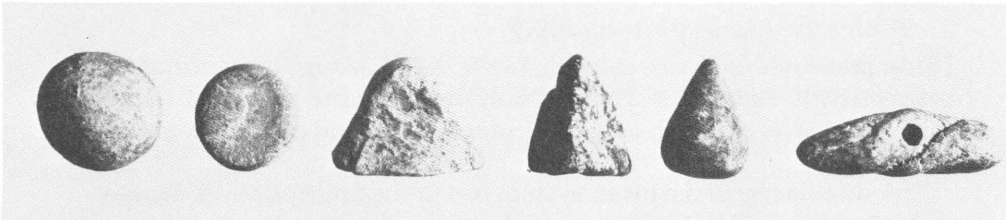
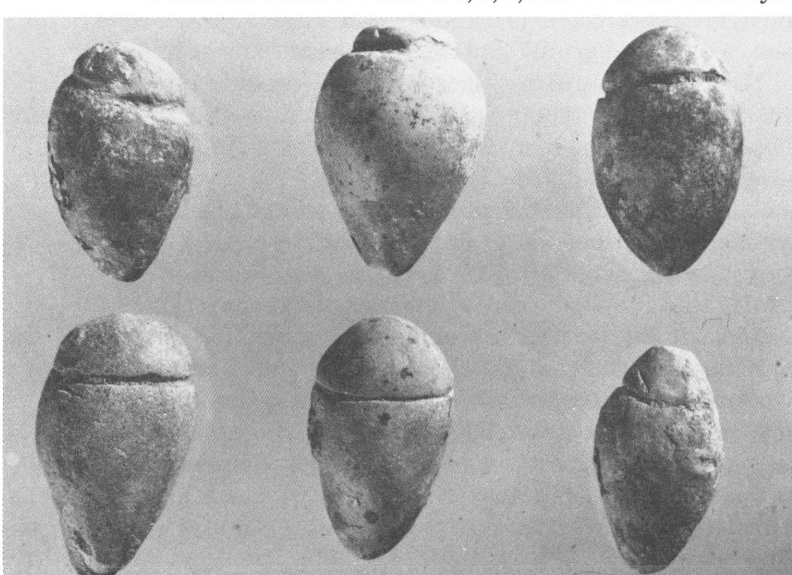


Figure 1. Tokens are small artifacts made of clay and shaped into various specific forms such as the sphere, disk, tetrahedrons, cone, and cylinder illustrated here. The tokens served as counters to keep track of goods in the ancient Middle East when writing had not yet been invented. Each token was a symbol representing one unit of a particular commodity. The cones, spheres, and disks, for example, probably stood for different measures of grain, whereas the tetrahedrons may have represented units of service and the twisted cylinder a bundle of rope.

Figure 2. Counting with tokens was performed in a one-to-one correspondence. Six jars of oil, for example, were represented by six ovoid tokens, as illustrated here, each ovoid standing for "one jar of oil." Each token merged together in a same symbol, therefore, the concept of the item counted and the concept of the number "one." This method of counting is known as "concrete counting." It is fundamentally different from "abstract counting" which expresses the concepts of oneness, twoness, threeness independently from the items counted. There were no abstract numerals such as 1, 2, 3, etc. . . . in the token system.



1 The counters were three-dimensional, tangible, and could be manipulated with the hand.

2 The token system fused together the notion of quality and two concepts of quantity. The ovoid which probably stands for "one jar of oil" merges together, for example, the concepts of "oil"; of quantity (how much) "jar"; and of number (how many) "one." This way of translating the visual image of the item is somewhat anologous to our concepts of "a keg of beer" and "a bottle of Chianti."

3 Most importantly, the tokens represented plurality as it is in nature: in one-to-one correspondence. Three jars of oil were expressed by one ovoid + one ovoid + one avoid which translated what three jars of oil are in reality: one jar of oil + one jar of oil + one jar of oil.

In sum, like the numerical expressions of concrete counting, the tokens did not abstract numbers. Each token merged the notions of (a) nature/quantity of a product; (b) the number *one.* This is why, like number words used in concrete counting, each token shape was specific to one item counted. Ovoids could only count jars of oil and jars of oil could only be counted with ovoids. Likewise, the cones could only count small measures of grain and large measures of grain could only be counted by spheres. Should we imagine what counting device would suit concrete counting, we would have to think in terms of a system, like that of the tokens, with different counters for expressing the different concrete numerations. It is conceivable, therefore, that the token system could reflect or derive from the practice of concrete counting. Like we count "one, two, three . . . " with the help of the beads of an abacus, the various types of tokens would have suggested the appropriate numeration to be used.

3 *From concrete to abstract counting*

When tokens were replaced by their image impressed on the surface of a clay tablet, a sphere, for instance, was replaced by a circular impressed mark and a cone by a conical impressed mark (Figure 3). These impressed marks — ideograms — could no longer be grasped in the hand and manipulated, like the tokens had been. In this sense, the impressed ideograms were removed one step further from the actual real good they represented. Semantically, however, the impressed marks were identical to the tokens. Each ideogram still fused together the concepts of (1) nature/quantity (i.e., measure of grain) and (2) the number *one.* At this stage plurality was still expressed in a one-to-one correspondence. For example, two measures of grain were shown by *two* conical marks. The impressed tablets, therefore, do not reflect any change in the counting practice.

At the next stage in the evolution of writing, which is characterized by the technique of tracing ideograms with the *sharp* end of a stylus rather than impressing them with the *blunt* end of a stylus, plurality is no longer expressed by one-to-one correspondence. The incised pictographs representing units of goods, such as "jar of oil," are no longer repeated according to the number of units in question. Three jars of oil, for example, are never indicated by repeating the unit "jar of oil" three times. Instead, the sign "jar of oil" is preceded by numerals — symbols expressing an abstract number. Notations expressing abstract numbers are first present, therefore, on the pictographic tablets of Uruk IVa, ca. 3100 B.C. This does not say that 3100 B.C. is the time when abstract numbers were conceived. It says that 3100 B.C. is the time when we observe a change in the record keeping technique.

Figure 3. The tablet displays two kinds of information. First, the all-over pattern showing various kinds of jars is the impression of a seal which identified an office or an individual. Second, the circular and conical marks are notations impressed with a stylus. These marks replaced the tokens after 3200 B.C. They still perpetuated the form of the tokens and stood for the same units of goods. For example, the circular and conical marks shown on this tablet stood for measures of grain. The marks, like the tokens, were used in a one-to-one correspondence. There were still no numerals to express abstract numbers.

It appears that the conical and circular impressed marks — which continued to be impressed while the other signs were being traced with the sharp end of a stylus — could be read, according to the context, either as units of grain or "one" and "six."[28] For example, on the tablet shown in Figure 4, the three circular marks and the three conical marks associated with the sign "jar of [oil/beer?]" are probably to be read as "18 jars of [oil/beer?]." It was a leap from the original concrete reading: a measure of grain, to a secondary abstract meaning: an abstract number. The choice of metrological units of grain for a more general use appears logical, first, because grain was the commodity most widely exchanged in the ancient Middle East. It played the role of currency and must have been, therefore, the most familiar accounting system. Second, the grain accounting system provided a

Figure 4. With the introduction of *incised* pictographs — rather than *impressed* signs — plurality is no longer represented by a one-to-one correspondence. The incised pictographs representing units of goods such as "a jar of oil" are no longer repeated according to the number of units in question. Instead, the sign "jar of oil" is preceded by numerals — symbols expressing an abstract number. Each conical mark is to be read as the numeral "1" and each circular mark as the numeral "6." The tablet thus records a total of "21 jars of oil." The cone and sphere which represented the most basic units of grain led, ultimately, therefore, to the development of numerals standing for the abstract numbers "1" and "6" in the Sumerian numerical system.

unique gamut of units which could be easily converted into a sequence of numerical units of growing magnitude. Further studies will be necessary to show how the various measures of grain became standardized to become multiples of one another, leading to such equation as 6 units (ban) equal 1 large unit (bariga). Thus, it appears that the cone and the sphere which probably represented the most basic quantities of grain handled in daily life ultimately led to the development of numerals standing for 1 and 6 in the Sumerian numerical system.

The system of notation was not fully abstract, however, and small numbers were still indicated by 2, 3, 4, 5 impressed conical marks. A new element of abstraction in notation was the use of the impressed circular mark for the number six. This created an economy of notation since eighteen could be represented as three circular and three conical marks.[29]

Conclusion

The archaeological evidence suggests an evolution from concrete to abstract counting in the ancient Middle East hence supporting the linguistic evidence. As is typical in concrete counting, the notions of the nature of the commodity and quantity (how many) were inseparable in the tokens used for counting between 8000-3100 B.C. Writing, which appears about 3100 B.C. first provided two parallel systems of notations which split the notions of quality and quantity (how many). The first system of notations were numerals (impressed marks) expressing abstract numbers and the second (incised ideograms) expressed the things counted. The new technology for record keeping appears to reflect, therefore, a radically new method of data processing with the use of abstract numerals. This is also supported by an abrupt reduction in the number of shapes of tokens about 3100 B.C.[30] It is assumed that the few remaining shapes, namely plain spheres and disks, were henceforth used as counters to calculate numerical amounts. The tokens would have no longer expressed concrete numbers.

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Figures 1, 2, 3, featuring respectively tokens from Susa, from Tello, and an impressed tablet from Susa has been provided by the Musée du Louvre, Département des Antiquités Orientales. Figure 4, showing a tablet from Godin Tepe (Gd.73.295) was obtained by courtesy of T. Cuyler Young, Jr., Royal Ontario Museum, Toronto. I want to express my appreciation to William W. Hallo who is currently engaged in a full study of the tablets of Godin Tepe for his permission to use this tablet. Table I illustrates the seven numerations used by the Tsimshians of British Columbia according to L. L. Conant, *The Number Concept,* MacMillan, New York, 1896.

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23. Jean Perrot, "Le Gisement Natoufien de Mallaha (Eynan), Israel," *L'Anthropologie, 70: 5-6,* 1966, fig. 22:26.

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25. Schmandt-Besserat (1983).

26. Denise Schmandt-Besserat, "The envelopes that bear the first writing," *Technology and Culture, 21: 3,* 1980, 371-375. The analysis of the groups of tokens found enclosed in envelopes supports the fact that until the last quarter of the fourth millennium B.C. tokens always represented units of goods — never numbers. As I have explained elsewhere, the tokens could not stand for numbers, because, if they had, the messages, or record, carried by the envelopes were useless. For instance, if the spheres stood for ten, as some have erroneously suggested, an envelope yielding three spheres would read: "18 of an unspecified product" or 18 x . Such imprecision is fully inconsistent with the idea of the token system which was obviously developed as a conscious effort to differentiate between the goods counted. It is fully inconsistent also with the bookeeping practice in the Sumerian bureaucracy. From the lists of goods featured on the tablets, we know that the accountants entered each product with painstaking precision, differentiating, for instance, between multiple kinds of breads, beers, or various breeds of sheep. The reading for three spheres as "three units of grain" is therefore logical. It is also impossible that some tokens represented goods and other numbers, as I had suggested earlier. The case of an envelope from Uruk makes it obvious. It contained, among others, 3 units of oil, 9 units of service, and 24 spheres. The only intelligible reading of these tokens is: 3 jars of oil, 9 units of service, and 24 bushels of grain. Otherwise it is impossible to know how many sixs referred to each item.

27. Schmandt-Besserat, (1980), p. 371.

28. Joran Friberg, "Numbers and measures in the earliest written records," *Scientific American, 250: 2,* 1984, 116. Friberg argues that the circular sign can be read either as 6 or 10.

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